OTFS: A New Modulation Scheme for 5G and Beyond

NCC’2020 Tutorial
IIT Kharagpur

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Special thanks to
M. K. Ramachandran, G. D. Surabhi, Rosemary Augustine, Prof. Emanuele Viterbo

21 February 2020
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2 High-mobility requirement in 5G

3 OTFS modulation

4 Performance of OTFS
   - Diversity in OTFS
   - MIMO-OTFS
   - Space-time coded OTFS
   - PAPR of OTFS
   - Channel estimation in OTFS
   - Multiuser OTFS (OTFS-MA)
   - Effect of phase noise on OTFS

5 Conclusions
Some background
Wireless spectrum

Source: Internet
Some wireless terminologies

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency (Hz)</td>
<td>The frequency of the signal in Hertz.</td>
</tr>
<tr>
<td>Bandwidth (Hz)</td>
<td>The range of frequencies over which the signal varies.</td>
</tr>
<tr>
<td>Data rate (bps)</td>
<td>The amount of data transmitted per second in bits.</td>
</tr>
<tr>
<td>Spectral efficiency (bps/Hz)</td>
<td>The amount of data transmitted per Hertz.</td>
</tr>
<tr>
<td>Signaling interval (Ts)</td>
<td>The time interval between signals.</td>
</tr>
<tr>
<td>Signal-to-noise ratio (SNR)</td>
<td>The ratio of the signal power to the noise power.</td>
</tr>
<tr>
<td>Channel capacity (bps)</td>
<td>The maximum amount of information that can be transmitted.</td>
</tr>
<tr>
<td>Probability of bit error</td>
<td>The likelihood of an error in a bit transmitted.</td>
</tr>
<tr>
<td>Multipath fading</td>
<td>The effect of multiple paths on the signal.</td>
</tr>
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</table>

OTFS: A New Modulation Scheme for 5G and Beyond
21 February, 2020
Some wireless terminologies

- Carrier frequency (Hertz; Hz)
Some wireless terminologies

- Carrier frequency (Hertz; Hz)
- Bandwidth, $W$ (Hz)
Some wireless terminologies

- Carrier frequency (Hertz; Hz)
- Bandwidth, $W$ (Hz)
Some wireless terminologies

- Carrier frequency (Hertz; $\text{Hz}$)
- Bandwidth, $W$ (Hz)
- Data rate (bits per second; $\text{bps}$)
Some wireless terminologies

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Some wireless terminologies

- Carrier frequency (Hertz; Hz)
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Some wireless terminologies

- **Carrier frequency (Hertz; Hz)**
- **Bandwidth, $W$ (Hz)**
- **Data rate (bits per second; bps)**
- **Spectral efficiency (bps per Hz; bps/Hz)**
- **Signaling interval, $T_s = \frac{1}{W}$ (sec)**
- **Signal-to-noise ratio (SNR)**
Some wireless terminologies

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- Probability of bit error
- Multipath fading
AWGN channel

\[ n(t) \quad s(t) \quad y(t) = s(t) + n(t) \]
AWGN channel

\[ y(t) = s(t) + n(t) \]

- e.g., satellite channel

\[ C = W \log(1 + \text{SNR}) \]

- \( P_e / P_0 \) vs. SNR
- Probability of error falls exponentially with SNR.
- Capacity grows with SNR (but only logarithmically with SNR).
AWGN channel

- e.g., satellite channel

\[
n(t)
\]
\[
s(t) + n(t)
\]
\[
y(t) = s(t) + n(t)
\]

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<td>$P_e \propto e^{-SNR}$</td>
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AWGN channel

\[ n(t) \]

\[ s(t) \rightarrow y(t) = s(t) + n(t) \]

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- Prob. of error \textit{falls exponentially} with SNR 😊
AWGN channel

e.g., satellite channel

\[
\begin{align*}
n(t) & \quad \rightarrow \quad n(t) \\
\downarrow & \quad \rightarrow \quad y(t) = s(t) + n(t)
\end{align*}
\]

- Prob. of error falls exponentially with SNR
- Capacity grows with SNR (but only logarithmically with SNR)

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Multipath fading

Variation in frequency-domain
- Max. Delay spread ($\text{max}$)
- Coherence bandwidth ($W_{\text{coh}}$)

Variation in time-domain
- Max. Doppler spread ($\text{max}$)
- Coherence time ($T_{\text{coh}}$)
Multipath fading

- **Fading channel characterization**
Multipath fading

- **Fading channel characterization**
  - **Variation in frequency-domain**
    - Max. Delay spread ($\tau_{\text{max}}$)
    - Coherence bandwidth ($W_{\text{coh}}$)
Multipath fading

Fading channel characterization

Variation in frequency-domain
- Max. Delay spread ($\tau_{max}$)
- Coherence bandwidth ($W_{coh}$)

Variation in time-domain
- Max. Doppler spread ($\nu_{max}$)
- Coherence time ($T_{coh}$)
Multipath fading

- **Variation in frequency-domain**
  - **Max. Delay spread** ($\tau_{\text{max}}$) and **coherence bandwidth** ($W_{\text{coh}}$)
Multipath fading

- Variation in frequency-domain
  - Max. Delay spread ($\tau_{\text{max}}$) and coherence bandwidth ($W_{\text{coh}}$)

[Diagram showing LoS path and reflected paths with labels $r_1$, $r_2$, $r_3$.]
Multipath fading

- **Variation in frequency-domain**
  - Max. Delay spread ($\tau_{max}$) and coherence bandwidth ($W_{coh}$)

- Delay of LoS path: $\tau_1 = r_1 / c$. 

\[ r_2 \quad r_3 \quad \text{Reflected path} \]
\[ r_1 \quad \text{LoS path} \]
Multipath fading

- Variation in frequency-domain

  - Max. Delay spread ($\tau_{max}$) and coherence bandwidth ($W_{coh}$)

  - Delay of LoS path: $\tau_1 = r_1/c$. Delay of reflected path: $\tau_2 = (r_2 + r_3)/c$
Multipath fading

- **Variation in frequency-domain**
  - Max. Delay spread ($\tau_{max}$) and coherence bandwidth ($W_{coh}$)

- Delay of LoS path: $\tau_1 = r_1/c$. Delay of reflected path: $\tau_2 = (r_2 + r_3)/c$
- Delay spread: $\tau_2 - \tau_1$. 
Multipath fading

Variation in frequency-domain

Max. Delay spread ($\tau_{\text{max}}$) and coherence bandwidth ($W_{\text{coh}}$)

Delay of LoS path: $\tau_1 = r_1/c$.

Delay spread: $\tau_2 - \tau_1$.

Delay of reflected path: $\tau_2 = (r_2 + r_3)/c$.

$\tau_{\text{max}} = \max_{i,j} |\tau_i - \tau_j|$.
Multipath fading

- **Variation in frequency-domain**
  
  - *Max. Delay spread* ($\tau_{\text{max}}$) and *coherence bandwidth* ($W_{\text{coh}}$)

  ![Diagram of multipath fading with LoS and reflected paths]

  - Delay of LoS path: $\tau_1 = \frac{r_1}{c}$
  - Delay spread: $\tau_2 - \tau_1$
  - $W_{\text{coh}} \propto \tau_{\text{max}}^{-1}$

  Delay of reflected path: $\tau_2 = \frac{(r_2 + r_3)}{c}$

  $\tau_{\text{max}} = \max_{i,j} |\tau_i - \tau_j|$
Multipath fading

- **Variation in frequency-domain**
  - **Max. Delay spread** $(\tau_{\text{max}})$ and **coherence bandwidth** $(W_{\text{coh}})$

- Delay of LoS path: $\tau_1 = r_1/c$.  
- Delay of reflected path: $\tau_2 = (r_2 + r_3)/c$
- Delay spread: $\tau_2 - \tau_1$.  
  
  $\tau_{\text{max}} = \max_{i,j} |\tau_i - \tau_j|$

- $W_{\text{coh}} \propto \tau_{\text{max}}^{-1}$

- **Frequency-flat fading**: Coherence BW $> \text{Signaling BW}$ $(W_{\text{coh}} > W)$
Multipath fading

**Variation in frequency-domain**

- **Max. Delay spread** ($\tau_{\text{max}}$) and **coherence bandwidth** ($W_{\text{coh}}$)

  - Delay of LoS path: $\tau_1 = r_1/c$.  
  - Delay of reflected path: $\tau_2 = (r_2 + r_3)/c$
  - Delay spread: $\tau_2 - \tau_1$.  
  - $\tau_{\text{max}} = \max_{i,j} |\tau_i - \tau_j|$
  
  - $W_{\text{coh}} \propto \tau_{\text{max}}^{-1}$

- **Frequency-flat fading**: Coherence BW > Signaling BW ($W_{\text{coh}} > W$)
- **Frequency-selective fading**: Coherence BW < Signaling BW ($W_{\text{coh}} < W$)
Multipath fading

- **Variation in frequency-domain**
  - **Max. Delay spread** ($\tau_{\text{max}}$) and **coherence bandwidth** ($W_{\text{coh}}$)

  \[
  \text{Delay of LoS path: } \tau_1 = \frac{r_1}{c}.
  \]
  \[
  \text{Delay of reflected path: } \tau_2 = \frac{(r_2 + r_3)}{c}.
  \]
  \[
  \text{Delay spread: } \tau_{\text{max}} = \max_{i,j} |\tau_i - \tau_j|.
  \]
  \[
  W_{\text{coh}} \propto \tau_{\text{max}}^{-1}
  \]

  - **Frequency-flat fading**: Coherence BW > Signaling BW ($W_{\text{coh}} > W$)
  - **Frequency-selective fading**: Coherence BW < Signaling BW ($W_{\text{coh}} < W$)
Variation in time-domain

Max. Doppler spread ($\nu_{max}$) and coherence time ($T_{coh}$)

Doppler shift of LoS path: $\nu_{max} = f_c v$

Doppler shift of reflected path: $\nu_{max} = f_c v \cos \theta$

Doppler spread: $\nu_{max}$

Slow fading: Coherence time $T_{coh} >$ signaling interval ($T_s$)

Fast fading: Coherence time $T_{coh} <$ signaling interval ($T_s$)
Multipath fading

- **Variation in time-domain**
  - Max. Doppler spread \((\nu_{max})\) and coherence time \((T_{coh})\)

- Doppler shift of LoS path: \(\nu_1 = \frac{v}{\lambda} = f_c \frac{v}{c}\)
Multipath fading

- **Variation in time-domain**
  - Max. Doppler spread ($\nu_{max}$) and coherence time ($T_{coh}$)

- Doppler shift of LoS path: $\nu_1 = \frac{v}{\lambda} = f_c \frac{v}{c}$
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Multipath fading

- Variation in time-domain
  - Max. Doppler spread \( (\nu_{\text{max}}) \) and coherence time \( (T_{coh}) \)

- Doppler shift of LoS path: \( \nu_1 = \frac{v}{\lambda} = f_c \frac{v}{c} \)
- Doppler shift of reflected path: \( \nu_2 = f_c \frac{v \cos \theta}{c} \)
- Doppler spread: \( \nu_2 - \nu_1 \)
Variation in time-domain

- Max. Doppler spread ($\nu_{\text{max}}$) and coherence time ($T_{\text{coh}}$)

- Doppler shift of LoS path: $\nu_1 = \frac{v}{\lambda} = f_c \frac{v}{c}$
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Multipath fading

- Variation in time-domain
  - Max. Doppler spread ($\nu_{\text{max}}$) and coherence time ($T_{\text{coh}}$)

### LoS path and Reflected path

- Doppler shift of LoS path: $\nu_1 = \frac{v}{\lambda} = f_c \frac{v}{c}$
- Doppler shift of reflected path: $\nu_2 = f_c \frac{v \cos \theta}{c}$
- Doppler spread: $\nu_2 - \nu_1 \quad \nu_{\text{max}} = \max_{i,j} |\nu_i - \nu_j|$
Multipath fading

- Variation in time-domain
  - Max. Doppler spread ($\nu_{\text{max}}$) and coherence time ($T_{\text{coh}}$)

- Doppler shift of LoS path: $\nu_1 = \frac{v}{\lambda} = \frac{f_c v}{c}$
- Doppler shift of reflected path: $\nu_2 = \frac{f_c v \cos \theta}{c}$
- Doppler spread: $\nu_2 - \nu_1$  
  \[ \nu_{\text{max}} = \max_{i,j} |\nu_i - \nu_j| \]
- $T_{\text{coh}} \propto \nu_{\text{max}}^{-1}$
Variation in time-domain

Max. Doppler spread $(\nu_{\text{max}})$ and coherence time $(T_{\text{coh}})$

- Doppler shift of LoS path: $\nu_1 = \frac{v}{\lambda} = f_c \frac{v}{c}$
- Doppler shift of reflected path: $\nu_2 = f_c \frac{v \cos \theta}{c}$
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$T_{\text{coh}} \propto \nu_{\text{max}}^{-1}$

- Slow fading: Coherence time $\geq$ signaling interval $(T_{\text{coh}} > T_s)$
Multipath fading

- Variation in time-domain
  - Max. Doppler spread \( (\nu_{\text{max}}) \) and coherence time \( (T_{\text{coh}}) \)

- Doppler shift of LoS path: \( \nu_1 = \frac{\nu}{\lambda} = f_c \frac{v}{c} \)
- Doppler shift of reflected path: \( \nu_2 = f_c \frac{v \cos \theta}{c} \)
- Doppler spread: \( \nu_2 - \nu_1 \quad \nu_{\text{max}} = \max_{i,j} |\nu_i - \nu_j| \)

- \( T_{\text{coh}} \propto \nu_{\text{max}}^{-1} \)
- Slow fading: Coherence time \( \geq \) signaling interval \( (T_{\text{coh}} > T_s) \)
- Fast fading: Coherence time \( \leq \) signaling interval \( (T_{\text{coh}} < T_s) \)
Fading channel

- e.g., mobile radio channel

\[
\begin{align*}
    s(t) & \quad \times \quad h(t) \quad \rightarrow \quad y(t) = h(t)s(t) + n(t) \\
    n(t) & \quad \rightarrow \quad y(t)
\end{align*}
\]

<table>
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<tr>
<th>Channel</th>
<th>Error Probability ((P_e))</th>
<th>Capacity ((C)), bps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fading</td>
<td>(P_e \propto SNR^{-1})</td>
<td>(C = W \log(1 + SNR))</td>
</tr>
</tbody>
</table>

- Prob. of error falls only linearly with SNR
Prob. of bit error performance

![Graph showing the probability of bit error performance against average SNR in dB for AWGN and Fading conditions. The graph includes a line for BPSK.]
ISI channel

- e.g., mobile radio channel

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<th>Channel</th>
<th>Error Probability ($P_e$)</th>
<th>Capacity ($C$), bps</th>
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<tbody>
<tr>
<td>ISI</td>
<td>$P_e \propto SNR^{-L}$</td>
<td>$C = W \log(1 + SNR)$</td>
</tr>
</tbody>
</table>

Causes inter-symbol interference (ISI)

If rx. signal is properly processed (equalization)

- prob. of error falls with $L$th power of SNR (multipath diversity)
OFDM in ISI channels

- **Orthogonal Frequency Division Multiplexing** *(overcomes ISI issue)*
**OFDM in ISI channels**

- **Orthogonal Frequency Division Multiplexing (overcomes ISI issue)**
- Converts frequency-selective channel to multiple \( \frac{M}{M_{1}} \) frequency-flat channels
OFDM in ISI channels

- **Orthogonal Frequency Division Multiplexing (overcomes ISI issue)**
- Converts frequency-selective channel to multiple \( M \) frequency-flat channels

\[
\text{Tx: } x(n) = \sum_{k=0}^{M-1} X(k)e^{j2\pi kn/M}, \quad n=0; 1; \ldots; M-1
\]

\[
\text{Rx: } Y(k) = H(k)X(k) + N(k), \quad k=0; 1; \ldots; M-1
\]
**OFDM in ISI channels**

- **Orthogonal Frequency Division Multiplexing (overcomes ISI issue)**
  - Converts frequency-selective channel to multiple \( M \) frequency-flat channels

- Multiplexes \( M \) information symbols \( X(k), k = 0, \cdots, M - 1 \) on \( M \) subcarriers
OFDM in ISI channels

- **Orthogonal Frequency Division Multiplexing** (overcomes ISI issue)
  - Converts frequency-selective channel to multiple \( M \) frequency-flat channels

- Multiplexes \( M \) information symbols \( X(k), k = 0, \cdots, M - 1 \) on \( M \) subcarriers

\[
\text{Rx: } Y(k) = \sum_{n=0}^{M-1} y(n) e^{j2\pi k n M}, k = 0, \cdots, M - 1
\]

\[
\text{Tx: } x(n) = \sum_{k=0}^{M-1} X(k) e^{-j2\pi k n M}, n = 0, \cdots, M - 1
\]

\[
X[k] \xrightarrow{\text{IFFT}} x[n] \xrightarrow{\text{ISI channel}} y[n] \xrightarrow{\text{FFT}} Y[k]
\]
OFDM in ISI channels

- **Orthogonal Frequency Division Multiplexing (overcomes ISI issue)**
  - Converts frequency-selective channel to multiple ($M$) frequency-flat channels

- Multiplexes $M$ information symbols $X(k), k = 0, \cdots, M - 1$ on $M$ subcarriers

- **Tx:** $x(n) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} X(k) e^{j2\pi \frac{kn}{M}}, \quad n = 0, 1, \cdots, M - 1$

---

**Diagram:**

- Channel with frequency-selective characteristics
- Subchannels after multiplexing
- Flowchart of OFDM process:
  - $X[k] \xrightarrow{\text{IFFT}} x[n] \xrightarrow{\text{ISI channel}} y[n] \xrightarrow{\text{FFT}} Y[k]$

---

Source: Internet
OFDM in ISI channels

- **Orthogonal Frequency Division Multiplexing (overcomes ISI issue)**
  - Converts frequency-selective channel to multiple ($M$) frequency-flat channels

- Multiplexes $M$ information symbols $X(k), k = 0, \cdots, M - 1$ on $M$ subcarriers

\[
\begin{align*}
\text{Tx: } x(n) &= \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} X(k) e^{j\frac{2\pi kn}{M}}, \quad n = 0, 1, \cdots, M - 1 \\
\text{Rx: } Y(k) &= \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} y(n) e^{-j\frac{2\pi kn}{M}}, \quad k = 0, 1, \cdots, M - 1
\end{align*}
\]
**Orthogonal Frequency Division Multiplexing (overcomes ISI issue)**

- Converts frequency-selective channel to multiple \( M \) frequency-flat channels

- Multiplexes \( M \) information symbols \( X(k), k = 0, \cdots, M-1 \) on \( M \) subcarriers

**Transmit (Tx):**

\[
x(n) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} X(k) e^{j2\pi kn/M}, \quad n = 0, 1, \cdots, M-1
\]

**Receive (Rx):**

\[
Y(k) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} y(n) e^{-j2\pi kn/M}, \quad k = 0, 1, \cdots, M-1
\]

- Renders the ISI channel to a multiplicative channel, \( Y(k) = H(k)X(k) + N(k) \)
**SIMO channel**

- e.g., mobile radio channel

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<td>SIMO</td>
<td>$P_e \propto SNR^{-n_r}$</td>
<td>$C = W \log(1 + SNR)$</td>
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- Prob. of error falls with $n_r$th power of SNR (receive diversity) 😊
MIMO channel

- e.g., mobile radio channel \( y = Hx + n \)

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<th>Capacity ((C)), bps</th>
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<tr>
<td>MIMO</td>
<td>( P_e \propto SNR^{-ntn_r} )</td>
<td>( C = \min(nt, nr) W \log(1 + SNR) )</td>
</tr>
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</table>

- Prob. of error falls with \( nt \cdot nr \)th power of SNR (tx & rx diversity) 😊
- Capacity grows linearly with \( nt, nr \) 😊
- Large \( nt, nr \) \(\implies\) large capacity/diversity gains (massive MIMO in 5G)
- MIMO signal detection problem: \( \hat{x} = \arg\max_{x \in A^{nt}} \| y - Hx \|^2 \)
MIMO
- spectrally efficient, reliable, power efficient
Multiuser communication

User 1

User 2

User 3

\ldots

User \(K\)

\begin{align*}
1 &\quad \quad \quad \quad \quad 1 \\
2 &\quad \quad \quad \quad \quad 2 \\
3 &\quad \quad \quad \quad \quad 3 \\
\vdots &\quad \quad \quad \quad \quad \vdots \\
N &\quad \quad \quad \quad \quad N \\
\end{align*}

\begin{align*}
\text{Base Station} \\
\quad \quad \\
\text{Uplink} \\
\quad \quad \\
\text{Downlink}
\end{align*}
# Cellular wireless evolution

<table>
<thead>
<tr>
<th>Generation</th>
<th>Frequency band</th>
<th>PHY features</th>
<th>Data rate</th>
<th>Spectral Eff. (bps/Hz)</th>
</tr>
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<tbody>
<tr>
<td>1G</td>
<td>850 MHz</td>
<td>FDMA, FM</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>2G</td>
<td>900 MHz, 1.8 GHz</td>
<td>TDMA/CDMA, GMSK/QPSK, FEC, PC</td>
<td>10 Kbps</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>3G</td>
<td>1.8–2.5 GHz</td>
<td>CDMA, QAM</td>
<td>1–40 Mbps</td>
<td>1–8</td>
</tr>
<tr>
<td>4G</td>
<td>2–8 GHz</td>
<td>OFDMA, SC-FDMA QAM, MIMO-OFDM</td>
<td>100–600 Mbps</td>
<td>15</td>
</tr>
<tr>
<td>5G</td>
<td>1–6 GHz mm wave (26–28 GHz) &lt; 1 GHz (massive IoT) visible light?</td>
<td>massive MIMO beamforming D2D, Full duplex, NOMA LDPC and Polar codes OFDM &amp; variants (adapted to extremes?)</td>
<td>multi-Gbps</td>
<td>several tens</td>
</tr>
</tbody>
</table>

- **Waveform design is the major change between the generations**
High-mobility requirement in 5G
Enhanced mobile broadband (eMBB) [2020-2021] (user focus)

- High data rates (multi-gigabits per sec)
- High spectral efficiency (tens of bps/Hz)
- High capacity (10 Tbps per Km²)
- High user mobility?
**5G vision**

1. **Enhanced mobile broadband (eMBB) [2020-2021] (user focus)**
   - High data rates (multi-gigabits per sec)
   - High spectral efficiency (tens of bps/Hz)
   - High capacity (10 Tbps per Km²)
   - High user mobility?

2. **Massive machine type communications (mMTC) [2021-2022] (device focus)**
   - Massive Internet of Things
   - High density (1 million nodes per Km²)
   - Ultra-low energy (10 years+ battery life)
   - Deep coverage (reach challenging locations)
5G vision

1. **Enhanced mobile broadband (eMBB) [2020-2021] (user focus)**
   - High data rates (multi-gigabits per sec)
   - High spectral efficiency (tens of bps/Hz)
   - High capacity (10 Tbps per Km²)
   - High user mobility?

2. **Massive machine type communications (mMTC) [2021-2022] (device focus)**
   - Massive Internet of Things
   - High density (1 million nodes per Km²)
   - Ultra-low energy (10 years+ battery life)
   - Deep coverage (reach challenging locations)

3. **Ultra-reliable and low-latency communications (uRLLC) [2024-2025]**
   - Mission-critical services
   - Ultra-low latency (< 1 msec end-to-end latency)
   - Ultra-high reliability (< 1 out of 100 million packets lost)
   - Strong security (health/financial/government)
High-mobility support

- Mobility support requirement
  - relative speed between the user and the network edge at which consistent user experience must be ensured

- Enhanced mobility support (~ 5-fold) compared to that in 4G

- Mobility on demand
  - ranging from very high mobility users (e.g., users in bullet trains, airplanes) to low mobility/stationary devices (e.g., smart meters)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>User experienced data rate</th>
<th>E2E latency</th>
<th>Mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile BB in vehicles (cars, trains)</td>
<td>DL: 50 Mbps UL: 25 Mbps</td>
<td>10 msec</td>
<td>on demand up to 500 km/h</td>
</tr>
<tr>
<td>Airplane connectivity</td>
<td>DL: 15 Mbps UL: 7.5 Mbps</td>
<td>10 msec</td>
<td>on demand up to 1000 km/h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Connection density</th>
<th>Traffic density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile BB in vehicles (cars, trains)</td>
<td>2000/km²</td>
<td>DL: 100 Gbps/km² (25 Gbps/train, 50 Mbps/car)</td>
</tr>
<tr>
<td></td>
<td>500 active users/train, 4 trains or 1 active user/car, 2000 cars</td>
<td>UL: 50 Gbps/km² (12.5 Gbps/train, 25 Mbps/car)</td>
</tr>
<tr>
<td>Airplane connectivity</td>
<td>80/plane</td>
<td>DL: 1.2 Gbps/plane 600 Mbps/plane</td>
</tr>
<tr>
<td></td>
<td>60 airplanes/18000 km²</td>
<td></td>
</tr>
</tbody>
</table>

**High Doppler wireless channels**

- **High Dopplers due to**
  - High mobility (e.g., bullet trains)

<table>
<thead>
<tr>
<th>Carrier frequency</th>
<th>UE speed</th>
<th>Doppler shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 GHz</td>
<td>27 kmph</td>
<td>100 Hz</td>
</tr>
<tr>
<td></td>
<td>100 kmph</td>
<td>370.3 Hz</td>
</tr>
<tr>
<td></td>
<td>500 kmph</td>
<td>1.851 KHz</td>
</tr>
</tbody>
</table>

- **High carrier frequencies (e.g., mmWave, 28 GHz)**

<table>
<thead>
<tr>
<th>Carrier frequency</th>
<th>UE speed</th>
<th>Doppler shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>28 GHz</td>
<td>27 kmph</td>
<td>700 Hz</td>
</tr>
<tr>
<td></td>
<td>54 kmph</td>
<td>1.4 KHz</td>
</tr>
</tbody>
</table>
OFDM (Modulation in 4G)

- OFDM
  - Information is signaled in the frequency domain
  - 1-D transform in frequency domain (IFFT/FFT)
  - Orthogonality among the subcarriers is the key
In presence of high Doppler, subcarriers lose orthogonality.

This results in inter-carrier interference (ICI).

Causes severe degradation in bit error performance for high Dopplers (error floors).

Channel estimation and equalization in high Doppler channels is difficult.
Time-frequency modulation

- Motivation: overcome the ICI effect
- $M$ frequency bins and $N$ time bins

Multiplex information symbols, $X(n; m)$, in the TF grid

Perform 2-D transform in the TF plane

**Tx:**

$$x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X(n; m) g(t - nT) e^{j2\pi mf(t - nT)}$$

**Rx:**

$$Y(n; m) = A(t; f) e^{j\frac{\pi}{2}rf(t_0 - t)} dt_0$$

Do not perform that well in high Doppler scenarios
Time-frequency modulation

- Bin both frequency and time axes (motivation: overcome the ICI effect)
Time-frequency modulation

- Bin both frequency and time axes (motivation: overcome the ICI effect)
- $M$ frequency bins and $N$ time bins

![Diagram showing OFDM and TFM with frequency and time axes.](image-url)
Time-frequency modulation

- Bin both frequency and time axes \(^{(\text{motivation: overcome the ICI effect})}\)
- \(M\) frequency bins and \(N\) time bins

- Multiplex \(MN\) information symbols, \(X(n, m)\), in the TF grid
Time-frequency modulation

- Bin both frequency and time axes (motivation: overcome the ICI effect)
- \( M \) frequency bins and \( N \) time bins

- Multiplex \( MN \) information symbols, \( X(n, m) \), in the TF grid

\[ X[n, m] \xrightarrow{\text{Heisenberg transform}} x(t) \xrightarrow{\text{Time-varying channel}} y(t) \xrightarrow{\text{Wigner transform}} Y[n, m] \]
Time-frequency modulation

- Bin both frequency and time axes (motivation: overcome the ICI effect)
- $M$ frequency bins and $N$ time bins

- Multiplex $MN$ information symbols, $X(n, m)$, in the TF grid

- Perform 2-D transform in the TF plane
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Perform 2-D transform in the TF plane

**Tx:**

\[
x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X(n, m) g_t(t - nT) e^{j2\pi m\Delta f(t - nT)}
\]
Time-frequency modulation

- Bin both frequency and time axes (motivation: overcome the ICI effect)
- \(M\) frequency bins and \(N\) time bins

- Multiplex \(MN\) information symbols, \(X(n, m)\), in the TF grid

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\[Tx: \quad x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X(n, m)g_t(t - nT)e^{j2\pi m\Delta f(t - nT)}\]

\[Rx: \quad Y(n, m) = A(t, f)|_{t=nT, f=m\Delta f}, \quad A(t, f) = \int y(t')g_r^*(t' - t)e^{-j2\pi f(t' - t)}dt'\]
Time-frequency modulation

- Bin both frequency and time axes *(motivation: overcome the ICI effect)*
- *M* frequency bins and *N* time bins

- Multiplex *MN* information symbols, *X(n, m)*, in the TF grid

- Perform 2-D transform in the TF plane

  - **Tx:** \( x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X(n, m) g_t(t - nT) e^{j2\pi m\Delta f(t-nT)} \)

  - **Rx:** \( Y(n, m) = A(t, f)|_{t=nT, f=m\Delta f}, \quad A(t, f) = \int y(t') g_r^*(t' - t) e^{-j2\pi f(t'-t)} dt' \)

- Do not perform that well in high Dopplers
OTFS modulation
Orthogonal Time Frequency Space (OTFS) modulation*

- A promising modulation scheme for doubly-selective channels
- Information is signaled in Delay-Doppler (DD) domain rather than in Time-Frequency (TF) domain

Why OTFS?

- OTFS Vs OFDM: A performance comparison

![BER comparison of OTFS and OFDM systems with MMSE detection for $f_c = 4$ GHz, $\Delta f = 15$ KHz, $M = 12$, $N = 7$, $P = 5$, $\nu_{max} = 1.85$ KHz, BPSK.]

Figure: BER comparison of OTFS and OFDM systems with MMSE detection for $f_c = 4$ GHz, $\Delta f = 15$ KHz, $M = 12$, $N = 7$, $P = 5$, $\nu_{max} = 1.85$ KHz, BPSK.
Key features of OTFS

- Channel response in DD domain is invariant (for a larger observation time than in TF representation) and compact
- Each symbol experiences nearly constant channel gain
- Converts the multiplicative action of channel into a 2D convolution interaction with transmitted symbols
Channel representation in DD domain

- Different representations can be used to model LTV multipath channel in time \((t)\), frequency \((f)\), delay \((\tau)\), and Doppler \((\nu)\) variables.

- Impulse response of a time-varying channel can be expressed as a function of:
  - time-frequency \(H(t, f)\)
  - time-delay \(g(t, \tau)\)
  - delay-Doppler \(h(\tau, \nu)\)
Different representations can be used to model LTV multipath channel in time \( t \), frequency \( f \), delay \( \tau \), and Doppler \( \nu \) variables.

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- Impulse response of a time-varying channel can be expressed as a function of:
  - time-frequency $H(t, f)$
  - time-delay $g(t, \tau)$
  - delay-Doppler $h(\tau, \nu)$

In time-frequency $H(t, f)$ and time-delay $h(t, \tau)$ representations:

- channel coefficients vary with time at a rate that depends on mobility and carrier frequency.
Time-frequency and delay-Doppler responses

- **DD domain impulse response** $h(\tau, \nu)$ is more compact
  - channel taps in DD representation correspond to a cluster of reflectors with specific delay and Doppler values
  - the delay and Doppler values depend on reflectors’ relative distance and relative velocity, respectively, with the transmitter and receiver
  - relative velocity and distance remain roughly constant for at least a few msecs
  - Hence channel in DD domain appears **time invariant for a longer duration**
  - DD representation results in a **sparse representation**

Channel in **time-frequency** $H(t, f)$ and **delay-Doppler** $h(\tau, \nu)$ domains
Example of a wireless channel in an urban multi-lane scenario illustrating the sparsity and slow variability of the channel in the DD representation.

\[ y(t) = \int_{\nu} \int_{\tau} h(\tau, \nu)x(t - \tau)e^{j2\pi\nu(t-\tau)}d\tau d\nu \]
A signal can be represented as a function of time, or as a function of frequency, or as a quasi-periodic function of delay and Doppler. These 3 representations are interchangeable by means of canonical transforms.

Using Zak transforms $Z_t(\phi)$ and $Z_f(\phi)$, symbols in DD domain can be converted into time and frequency domains.
Implemented using simple pre- and post-processing (ISFFT & SFFT) over any multicarrier modulation scheme such as OFDM.

Figure: OTFS block diagram
Signaling in delay-Doppler domain

- The information symbols in DD domain $x[k, l]$s are mapped to TF symbols $X[n, m]$ using ISFFT as

$$X[n, m] = \frac{1}{MN} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k, l] e^{j2\pi \left( \frac{nk}{N} - \frac{ml}{M} \right)}$$

- TF plane is sampled at intervals $T$ and $\Delta f$, to obtain a 2D grid

$$\Lambda = \{(nT, m\Delta f), n = 0, \cdots, N - 1, m = 0, \cdots, M - 1\}$$

- Delay-Doppler grid $\Gamma$, reciprocal to $\Lambda$

$$\Gamma = \{\left(\frac{k}{NT}, \frac{l}{M\Delta f}\right), k = 0, \cdots, N - 1, l = 0, \cdots, M - 1\}$$
Signaling in delay-Doppler domain

- A 2D ISFFT translates every point on the DD plane into a corresponding basis function that covers the entire TF plane (2D orthogonal complex exponentials)

- OTFS basis functions (waveforms) have strong resilience to delay-Doppler shifts imparted by the channel (2D localized pulses in the DD domain)
OTFS basis functions

- $M = N = 32, \Delta f = 15$ KHz

Figure: OTFS basis functions in delay-Doppler domain, time-frequency domain, and time domain for Doppler index $k = 0$ and delay index $l = 0$. 
OTFS basis functions

- \( M = N = 32, \Delta f = 15 \text{ KHz} \)

Figure: OTFS basis functions in delay-Doppler domain, time-frequency domain, and time domain for Doppler index \( k = 2 \) and delay index \( l = 0 \).
OTFS basis functions

- $M = N = 32$, $\Delta f = 15$ KHz

![Diagram showing OTFS basis functions in delay-Doppler domain, time-frequency domain, and time domain for Doppler index $k = 2$ and delay index $l = 2$.]

**Figure**: OTFS basis functions in delay-Doppler domain, time-frequency domain, and time domain for Doppler index $k = 2$ and delay index $l = 2$. 
OTFS basis functions

Figure: OTFS basis functions in time domain for \((k, l) = (0, 0), (k, l) = (0, 15), \) and \((k, l) = (2, 0).\)

- as Doppler index \((k)\) changes, frequency of pulse train changes (as in FDM)
- as delay index \((l)\) changes, position of pulses gets shifted in time (as in TDM)
Time-frequency domain (inner block)

- **Modulator**: Heisenberg transform

\[
x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X[n, m] g_{tx}(t - nT) e^{j2\pi m \Delta f (t-nT)}
\]

- **Channel**: \( h(\tau, \nu) \)

\[
y(t) = \int_{\nu} \int_{\tau} h(\tau, \nu) x(t - \tau) e^{j2\pi \nu (t-\tau)} d\tau d\nu
\]

- **Demodulator**: Wigner Transform

\[
A_{g_{rx}, y}(\tau, \nu) = \int g_{rx}^*(t - \tau) y(t) e^{-j2\pi \nu (t-\tau)} dt
\]

\[
Y[n, m] = A_{g_{rx}, y}(\tau, \nu) |_{\tau=nT, \nu=m\Delta f}
\]
If $h(\tau, \nu)$ is bounded by $(\tau_{\text{max}}, \nu_{\text{max}})$, s.t $\nu_{\text{max}} < \Delta f < 1/\tau_{\text{max}}$, then

$$Y[n, m] = H[n, m]X[n, m] + V[n, m],$$

$$H[n, m] = \int_{\tau} \int_{\nu} h(\tau, \nu)e^{j2\pi n\nu \tau}e^{-j2\pi(\nu + m\Delta f)\nu}d\nu d\tau$$

SFFT is then applied to $Y[n, m]$ to convert it back to delay-Doppler domain to obtain $y[k, l]$ as

$$y[k, l] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} Y[n, m]e^{-j2\pi\left(\frac{n}{N} - \frac{m}{M}\right)}$$
Input-output relation

For a channel with $P$ paths/cluster of reflectors

$$h(\tau, \nu) = \sum_{i=1}^{P} h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i)$$

Received signal in delay-Doppler domain: $\tau_i = \frac{\alpha_i}{M\Delta f}$ and $\nu_i = \frac{\beta_i}{N\Delta T}$

$$y[k, l] = \sum_{i=1}^{P} h_i e^{-j2\pi \nu_i \tau_i} x[(k - \beta_i)_N, (l - \alpha_i)_M] + v[k, l]$$

$$= h[k, l] \odot x[k, l] \quad \text{(2D Circular Convolution)}$$

Vectorized formulation: Input-output relation can be vectorized as

$$y = Hx + v,$$

where $x, y, v \in \mathbb{C}^{MN \times 1}$, $H \in \mathbb{C}^{MN \times MN}$, $x_{k+Nl} = x[k, l]$. 
Performance of OTFS
Simulation parameters I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency (GHz)</td>
<td>4</td>
</tr>
<tr>
<td>Subcarrier spacing (kHz)</td>
<td>15</td>
</tr>
<tr>
<td>Frame size ((M, N))</td>
<td>((12, 7))</td>
</tr>
<tr>
<td>Number of paths ((P))</td>
<td>5</td>
</tr>
<tr>
<td>Delay profile</td>
<td>Exponential power-delay profile</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>500</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
</tbody>
</table>

Smallest resource block used in LTE: \(M = 12, N = 7\)
OTFS performance

**Figure**: Performance of OTFS and OFDM

\[ f_c = 4 \text{ GHz}, \Delta f = 15 \text{ kHz} \]

\[ M = 12, \ N = 7, \ P = 5 \]

BPSK, MMSE detection
## Simulation parameters II: IEEE 802.11p (WAVE)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency (GHz)</td>
<td>5.9</td>
</tr>
<tr>
<td>Subcarrier spacing (MHz)</td>
<td>0.156</td>
</tr>
<tr>
<td>Frame size ($M, N$)</td>
<td>(64, 12)</td>
</tr>
<tr>
<td>Number of paths ($P$)</td>
<td>8</td>
</tr>
<tr>
<td>Delay profile</td>
<td>Exponential power-delay profile</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>220</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
</tbody>
</table>
Figure: Performance of OTFS and OFDM

$f_c = 5.9 \text{ GHz}$, $\Delta f = 0.156 \text{ MHz}$

$M = 64$, $N = 12$, $P = 8$

BPSK, MMSE detection
Diversity in OTFS
Diversity performance of OTFS

- There are only $P$ non-zero elements in each row and column of the equivalent channel matrix ($H$).
- Input-output relation can be rewritten in an alternate form as

$$y^T = h'X + v^T,$$

where $h'$ is a $1 \times P$ vector whose $i$th entry is given by $h'_i = h_i e^{-j2\pi \nu_i \tau_i}$, and $X_{P \times MN}$ is the symbol matrix whose $i$th column ($i = k + Ni$, $i = 0, 1, \cdots, MN - 1$) is given by

$$X[i] = \begin{bmatrix}
X(k-\beta_1)N+N(\alpha_1)M \\
X(k-\beta_2)N+N(\alpha_2)M \\
\vdots \\
X(k-\beta_P)N+N(\alpha_P)M
\end{bmatrix}$$

Diversity performance of OTFS

- Pairwise error probability (PEP) between $X_i$ and $X_j$

$$P(X_i \rightarrow X_j|h', X_i) = Q\left(\sqrt{\frac{||h'(X_i - X_j)||^2}{2N_0}}\right)$$

- PEP averaged over the channel statistics can be upper bounded as

$$P(X_i \rightarrow X_j) \leq \frac{1}{\gamma^r \prod_{i=1}^{r} \frac{\lambda_i^2}{4P}}$$

where $\lambda_i$ is the $i$th non-zero singular value of the difference matrix $\Delta_{ij} = (X_i - X_j)$, $r$ is the rank of $\Delta_{ij}$, and $\gamma$ is the SNR.

- Diversity order

$$\rho_{\text{siso-otfs}} = \min_{i,j \neq i} \text{rank}(\Delta_{ij})$$

which is one.
Lower bound on BER

- BER can be bounded as

\[ \text{BER} \geq \frac{1}{2^{MN}} \sum_{k=1}^{\kappa} P(X_i \rightarrow X_j), \]

where \( \kappa \) denotes the number of \( \Delta_{ij} \)'s having rank one

- Assuming BPSK symbols,

\[ P(X_i \rightarrow X_j) = \mathbb{E} \left[ Q \left( \sqrt{2\gamma PMN|\tilde{h}_1|^2} \right) \right] \]

- At high SNRs,

\[ \text{BER} \geq \frac{\kappa}{2^{MN}} \frac{1}{4\gamma MN} \]

- As the values \( M \) and \( N \) increase, the \( 2^{MN} \) term can dominate the ratio \( \frac{\kappa}{2^{MN}} \)

- BER can decrease with a higher slope for higher frame sizes before meeting the diversity one lower bound

\(^1\text{When } P = MN, \kappa = 8 \ \forall \ MN\)
Diversity results

Simulation parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency (GHz)</td>
<td>4</td>
</tr>
<tr>
<td>Subcarrier spacing (kHz)</td>
<td>3.75</td>
</tr>
<tr>
<td>Number of paths ($P$)</td>
<td>4</td>
</tr>
<tr>
<td>Delay-Doppler profile ($\tau_i, \nu_i$)</td>
<td>$(0, 0), (0, \frac{1}{NT}), (\frac{1}{MT}, 0), (\frac{1}{MT}, \frac{1}{NT})$</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
</tbody>
</table>

Figure: BER performance of OTFS for (i) $M = 2, N = 2$, (ii) $M = 4, N = 2$, (iii) $M = 4, N = 4$. 
Note that the $\frac{K}{2^{MN}}$ values for the considered systems are $\frac{8}{16}$, $\frac{8}{256}$, and $\frac{8}{65536}$, respectively.

Though the asymptotic diversity order is one, increasing the frame size can lead to higher slopes (> 1) for BER curves in the finite SNR regime.

**Choice of M & N**

- M decides the delay resolution and $\nu_{\text{max}} < \frac{B}{M} = \Delta f < 1/\tau_{\text{max}}$.  
- N decides Doppler resolution and latency ($T_l = NT$).

**Example:** For $\tau_{\text{max}} = 1 \mu s$, $\nu_{\text{max}} = 5$ kHz, $B = 10$ MHz, and $T_l = 1$ ms,

- $\nu_{\text{max}} < \Delta f < 1/\tau_{\text{max}} \implies 5$ KHz $< \Delta f < 1$ MHz  
- $\Delta f$ can be chosen to be 20 kHz ($\implies M = \frac{10 \times 10^6}{20 \times 10^3} = 500$)  
- $T_l = NT = \frac{N}{\Delta f}$; $\implies N = T_l \Delta f = 1 \times 10^{-3} \times 20 \times 10^3 = 20$  
- $(M, N)$ can be chosen as (500,20)
Phase rotation for full diversity

Let 

\[ \Phi = \text{diag}\left\{ \phi_0^{(0)}, \cdots, \phi_{N-1}^{(0)}, \phi_0^{(1)}, \cdots, \phi_{N-1}^{(1)}, \cdots, \phi_{N-1}^{(M-1)} \right\} \]

be the phase rotation matrix and

\[ x' = \Phi x \]

be the phase rotated OTFS transmit vector.

OTFS with the above phase rotation achieves the full diversity of \( P \) when 

\[ \phi_q^{(l)} = e^{ja_q^{(l)}}, \quad q = 0, \cdots, N - 1, \quad l = 0, \cdots, M - 1 \]

are transcendental numbers with \( a_q^{(l)} \) real, distinct, and algebraic.
Full diversity using phase rotation

\[ \Phi = \text{diag}\{1, e^{j \frac{1}{MN}}, \ldots, e^{j \frac{MN-1}{MN}}\} \]

**Figure**: BER performance of OTFS without and with phase rotation for
1. \( M = N = 2 \),
2. \( M = 4, N = 2 \), and
3. \( M = N = 4 \), and BPSK.

**Figure**: BER performance of OTFS without and with phase rotation, \( M = N = 2 \), and 8-QAM.
MIMO-OTFS
\[ y_{\text{MIMO}} = H_{\text{MIMO}} x_{\text{MIMO}} + v_{\text{MIMO}} \]

Delay and Doppler models

<table>
<thead>
<tr>
<th>Path index ($i$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay ($\tau_i, \mu$s)</td>
<td>2.1</td>
<td>4.2</td>
<td>6.3</td>
<td>8.4</td>
<td>10.4</td>
</tr>
<tr>
<td>Doppler ($\nu_i, \text{Hz}$)</td>
<td>0</td>
<td>470</td>
<td>940</td>
<td>1410</td>
<td>1880</td>
</tr>
</tbody>
</table>

Simulation parameters

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</tr>
<tr>
<td>Frame size $(M, N)$</td>
<td>(32, 32)</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
<tr>
<td>MIMO configuration</td>
<td>$1 \times 1, 1 \times 2, 1 \times 3, 2 \times 2, 3 \times 3, 2 \times 3$</td>
</tr>
<tr>
<td>Maximum speed (kmph)</td>
<td>507.6</td>
</tr>
</tbody>
</table>
MIMO-OTFS performance

Figure: BER performance comparison between MIMO-OTFS and MIMO-OFDM

2 x 2 system, $f_o=4$ GHz, $M=32$, $N=32$, $\Delta f=15$ kHz, $P=5$
Diversity of MIMO-OTFS

- Each row of $H_{\text{MIMO}}$ has only $n_t P$ non-zero elements and each column has only $n_r P$ non-zero elements. The MIMO-OTFS system model can be written as

$$\begin{bmatrix}
  y_1^T \\
  y_2^T \\
  \vdots \\
  y_{n_r}^T
\end{bmatrix} = \begin{bmatrix}
  h'_{11} & h'_{12} & \cdots & h'_{1n_t} \\
  h'_{21} & h'_{22} & \cdots & h'_{2n_t} \\
  \vdots & \vdots & \ddots & \vdots \\
  h'_{n_r1} & h'_{n_r2} & \cdots & h'_{n_rn_t}
\end{bmatrix} \begin{bmatrix}
  X_1^T \\
  X_2^T \\
  \vdots \\
  X_{n_t}^T
\end{bmatrix} + \begin{bmatrix}
  v_1^T \\
  v_2^T \\
  \vdots \\
  v_{n_r}^T
\end{bmatrix},$$

- Diversity order achieved by MIMO-OTFS can be derived as

$$\rho_{\text{mimo-otfs}} = n_r \cdot \min_{i, j, i \neq j, k} \text{rank}(\Delta_{k,ij}) = n_r$$

- If $MN \times 1$ OTFS transmit vector from each antenna is multiplied by the phase rotation matrix $\Phi$, then diversity order achieved by phase rotated MIMO-OTFS system is equal to $Pn_r$
Diversity results in MIMO-OTFS

Figure: BER performance of 1 × 1 SISO-OTFS and 2 × 2 MIMO-OTFS systems.

Figure: BER performance of 1 × 2 OTFS system with i) $M = N = 2$ and ii) $M = 4$, $N = 2$. 
Figure: BER performance of $2 \times 2$ MIMO-OTFS system without and with phase rotation, $M = N = 2$. 

- $f_c = 4$ GHz, $\Delta f = 3.75$ kHz
- $n_t = n_r = 2$, $M = N = 2$
- $P = 4$, BPSK
- ML detection
Space-time coded OTFS
Use of space-time coding in OTFS

structure of Alamouti code generalized to matrices

Consider the alternate form of input-output relation for SISO-OTFS

\[ y^T = h'X + v^T, \]

where \( X \) is an \( MN \times MN \) symbol matrix.

Consider a space-time code (rate-1) using the Alamouti code structure (generalized to matrices) quasi-static DD channel over $T' (= 2)$ frames

$$
\tilde{X}_{ntMN \times T'MN} = \tilde{X}_{2MN \times 2MN} = \begin{bmatrix} X_1 & -X_2^H \\ X_2 & X_1^H \end{bmatrix}
$$

Corresponding OTFS transmit vectors, $k \in \{1, 2\}$

$$
X_k \leftrightarrow x_k, \quad X_k^H \leftrightarrow (\hat{x}_k)^* = (Px_k)^*.
$$

where

$$
P = P'_M \otimes P'_N.
$$

Transmitted OTFS vectors in the second frame duration are not just the conjugated vectors, but are conjugated and permuted vectors of those transmitted in the first frame duration.
Decoding

- Received vectors in the first and second frame duration are

\[
\begin{align*}
\mathbf{y}_1 &= \mathbf{H}_1 \mathbf{x}_1 + \mathbf{H}_2 \mathbf{x}_2 + \mathbf{v}_1 \\
\mathbf{y}_2 &= -\mathbf{H}_1 (\hat{\mathbf{x}}_2)^* + \mathbf{H}_2 (\hat{\mathbf{x}}_1)^* + \mathbf{v}_2.
\end{align*}
\]

- Permutation (\(\mathbf{P}\)) and conjugation are applied on \(\mathbf{y}_2\).

\[
\begin{bmatrix}
\mathbf{y}_1^* \\
(\hat{\mathbf{y}}_2)^*
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{H}_1 & \mathbf{H}_2 \\
\mathbf{H}_2^H & -\mathbf{H}_1^H
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_1 \\
\mathbf{x}_2
\end{bmatrix} + 
\begin{bmatrix}
\mathbf{v}_1 \\
(\hat{\mathbf{v}}_2)^*
\end{bmatrix}
\]

- Since two block columns of \(\hat{\mathbf{H}}\) are orthogonal, the decoding problem for \(\mathbf{x}_1\) and \(\mathbf{x}_2\) can be decomposed into two separate orthogonal problems.
Diversity

- Let $\Delta_{ij} \triangleq \tilde{X}_i - \tilde{X}_j$ be the difference matrix

\[
\rho_{\text{STC-OTFS}} = \min \{ \min_{i,j \neq i} \text{rank}(\Delta_{ij}), 2P \}.
\]

- $\text{rank}(\Delta_{ij})$ can be simplified as

\[
\text{rank}(\Delta_{ij}^H \Delta_{ij}) = 2 \times \text{rank}(\Delta_{1,ij}^H \Delta_{1,ij} + \Delta_{2,ij}^H \Delta_{2,ij}).
\]

whose min rank is two.

- STC-OTFS for $2 \times n_r$ can achieve an asymptotic diversity order of $2n_r$.

- With phase rotation applied to each of $n_t$ transmit antennas during every frame duration can yield a full delay Doppler space diversity of $2Pn_r$. 
Figure: BER performance of 

i) $1 \times 1$ OTFS, ii) $2 \times 1$ STC-OTFS, and iii) $2 \times 2$ STC-OTFS for $M = N = 2$.
Phase rotation in STC-OTFS

Figure: BER performance of $2 \times 1$ STC-OTFS with and without phase rotation for (i) $P = 2$ and (ii) $P = 4$.

STC-OTFS with phase rotation achieves full spatial and delay-Doppler diversity even for small frame sizes.
PAPR of OTFS
High PAPR is one of the key detrimental aspects in OFDM systems

- Max. PAPR in OFDM increases with \( M \), no. of subcarriers due to \( M \)-point IDFT operation at transmitter

PAPR of OTFS waveform is of interest

- In OTFS, max. PAPR grows linearly with \( N \) (and not with \( M \))


PAPR of OTFS transmit signal

- Time domain OTFS signal is given by

\[ s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X[n, m] g_{tx}(t - nT) e^{j2\pi m\Delta f(t-nT)} \]

- Discrete time representation (by Nyquist sampling, \( t = (r + qM) T_s \)):

\[ s(r + qM) = M \sum_{n=0}^{N-1} \tilde{x}_r[n] g_{tx}([r + qM - nM]_{MN}) \]

where \( \tilde{x}_r[n] = \sum_{k=0}^{N-1} x[k, r] e^{j2\pi nk/N} \) (n\text{th} IDFT), \( r = 0, \cdots, M - 1 \) and \( q = 0, \cdots, N - 1 \)
Upper bound on PAPR

\[ \text{PAPR} = \max_{r, q} \left\{ |s(r + qM)|^2 \right\} / P_{\text{avg}}, \]

where

\[ P_{\text{avg}} = \frac{M^2 N \sigma_a^2}{MN} \sum_{n=0}^{N-1} \sum_{r=0}^{M-1} \sum_{q=0}^{N-1} |g_{tx}([r + qM - nM]_{MN})|^2 \]

\[ \max_{r, q} |s(r + qM)|^2 \leq M^2 N^2 \max_{k, l} |x[k, l]|^2 \max_{r, q} \sum_{n=0}^{N-1} |g_{tx}([r + qM - nM]_{MN})|^2 \]

For rectangular pulse,

\[ \text{PAPR}_{\text{max}} = \frac{N \max_c |c|^2}{\sigma_a^2} \]

\( \text{PAPR}_{\text{max}} \) grows linearly with \( N \) and not with \( M \).
Time domain samples of the OTFS transmit signal with rectangular pulse are the $N$-point IDFT values of the DD symbols along Doppler domain.

If $N$ is large, then by CLT, the transmitted samples can be approximated to have complex Gaussian distribution with zero mean.

CCDF of PAPR can be derived as

$$P(\text{PAPR} > \gamma_0) \approx 1 - (1 - e^{-\gamma_0})^{MN}$$
PAPR performance of OTFS

**Figure**: Analytical and simulated CCDF of PAPR in OTFS.

**Figure**: Effect of $M$ and $N$ on the CCDF of PAPR in OTFS.
OTFS can have better PAPR compared to OFDM and GFDM when $N < M$. 

Figure: CCDF of the PAPR of OTFS for different pulse shapes.

Figure: Comparison of CCDF of the PAPR of OTFS with those of OFDM and GFDM with 16-QAM.
Channel estimation in OTFS
Channel estimation in the delay-Doppler domain

- Each tx and rx antenna pair sees a different channel having a finite support in the delay-Doppler domain.
- The support is determined by the delay and Doppler spread of the channel.
- Input-output relation for pth tx and qth rx antenna pair can be written as

\[ \hat{x}_q[k, l] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x_p[n, m] \frac{1}{MN} h_{wp} \left( \frac{k - n}{NT}, \frac{l - m}{MΔf} \right) + v_q[k, l] \]

- Transmit pilot from pth antenna

\[ x_p[n, m] = 1 \text{ if } (n, m) = (n_p, m_p) \]
\[ = 0 \text{ if } (n, m) \neq (n_p, m_p) \]

- Received signal at the qth rx antenna will be

\[ \hat{x}_q[k, l] = \frac{1}{MN} h_{wp} \left( \frac{k - n_p}{NT}, \frac{l - m_p}{MΔf} \right) + v_q[k, l] \]

\[ \frac{1}{MN} h_{wp} \left( \frac{k}{NT}, \frac{l}{MΔf} \right) \] and thus \( \hat{H}_{qp} \) can be estimated, since \( n_p \) and \( m_p \) are known at the receiver a priori.
Figure: Illustration of pilots and channel response in delay-Doppler domain in a $2 \times 1$ MIMO-OTFS system

2 × 1 system, SNR= 4 dB
MIMO-OTFS performance with the estimated channel

Figure: Frobenius norm of the difference matrix $H_{\text{MIMO}} - \hat{H}_{\text{MIMO}}$ as a function of pilot SNR in a $2 \times 2$ MIMO-OTFS system

Figure: BER performance of $2 \times 2$ MIMO-OTFS system using the estimated channel
Multiuser OTFS (OTFS-MA)
Multiuser OTFS


OTFS-MA in the delay-Doppler domain

- Bins in the delay-Doppler grid $\Gamma$ serve as the delay-Doppler resource blocks (DDRBs)
- Different DDRBs are allocated to different users for multiple access
- Denoting the OTFS symbol vector transmitted by $u$th user by $x_u \in \mathbb{C}^{MN \times 1}$

$$y = \sum_{u=0}^{K_u-1} H_u x_u + v$$

Figure: OTFS multiple access (OTFS-MA) on the uplink
DDRB allocation schemes

Scheme 1- Multiplexing along delay axis

![Scheme 1 Diagram]

Scheme 2- Multiplexing along Doppler axis

![Scheme 2 Diagram]

Figure: DDRB allocation in an \( N \times M \) delay-Doppler grid in Scheme 1.

Figure: DDRB allocation in an \( N \times M \) delay-Doppler grid in Scheme 2.

The 2D circular convolution operation with channel results in each user’s symbols to experience MUI, requiring the BS to jointly decode the symbols.
**DDRB allocation schemes**

**Scheme 3 - MUI-free allocation**

![Diagram showing DDRB allocation in an N x M delay-Doppler grid in Scheme 3.](image)

- $MN/K_u$ symbols from a given user are placed at equal intervals in the delay as well as Doppler domains ($g_1$ and $g_2$ respectively where $K_u = g_1g_2$).
- $X_u[n,m]$ can be restricted to a region $\left[ \frac{NT}{g_2}(u)g_2, \frac{NT}{g_2}((u)g_2 + 1) \right]$ in time and $\left[ \left\lfloor \frac{M}{g_1} \right\rfloor \Delta f, \frac{M}{g_1} \left( \left\lfloor \frac{u}{g_2} \right\rfloor + 1 \right) \Delta f \right]$ in frequency.
- Reduced detection complexity.

---

Figure: BER performance of uplink OTFS-MA with different DDRB allocation schemes with $M = N = 4$, $K_u = 2, 4$, and ML detection $^1$.

$^1$4 tap channel, $\tau_{u,i} = \{0, 16.6, 33.3, 50\} \mu s$, $\nu_{\text{max}} = 1 \text{ KHz}$ $\forall u$

Figure: BER performance of uplink OTFS-MA with DDRB allocation Schemes 1 and 3 with $K_u = 4, 8$ users, $M = 64$, $N = 16$, and MP detection $^2$

$^2$10 tap channel, $\tau_{u,i} = \{0, 1.04, 2.08, 3.12, 4.16, 5.2, 6.25, 7.29, 8.33, 9.37\} \mu s$, $\nu_{\text{max}} = 1 \text{ KHz}$ $\forall u$. 

$f_c = 4 \text{ GHz}, \Delta f = 15 \text{ kHz}$

$M = N = 4$, $K_u = 2$

$\nu_{\text{max}} = 1 \text{ kHz}$, BPSK

ML detection

$K_u = 4$

$K_u = 4$

$K_u = 4$

$K_u = 4$

7
OTFS-MA Vs SC-FDMA and OFDMA

**Figure**: BER performance comparison between OTFS-MA, OFDMA, and SC-FDMA with ML detection.

**Figure**: BER performance comparison between OTFS-MA, OFDMA, and SC-FDMA with MP detection.

- **OTFS-MA** achieves better performance compared to OFDMA and SC-FDMA on the uplink.
OTFS with phase noise
Phase noise spectrum

Figure: PSD of oscillator phase noise for various carrier frequencies (4 GHz, 28 GHz, 60 GHz).

\[ L(f_m) = \frac{B_{PLL}^2}{B_{PLL}^2 + f_m^2} + L_{floor} \]

PLL bandwidth and variance of phase noise

![Graph showing variance of the oscillator phase noise as a function of $n$ where $B_{PLL} = n\Delta f$.]

Figure: Variance of the oscillator phase noise as a function of $n$ where $B_{PLL} = n\Delta f$. 

- $f_c = 4 \text{ GHz}, L_0 = -100 \text{ dBC/Hz}, L_{floor} = -158 \text{ dBC/Hz}$
- $f_c = 28 \text{ GHz}, L_0 = -80 \text{ dBC/Hz}, L_{floor} = -140 \text{ dBC/Hz}$
- $f_c = 60 \text{ GHz}, L_0 = -74 \text{ dBC/Hz}, L_{floor} = -132 \text{ dBC/Hz}$

$$B_{PLL} = n\Delta f$$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency (GHz)</td>
<td>28</td>
</tr>
<tr>
<td>Bandwidth (MHz)</td>
<td>10</td>
</tr>
<tr>
<td>Subcarrier spacing, $\Delta f$ (kHz)</td>
<td>78.125</td>
</tr>
<tr>
<td>Frame size ($M$, $N$)</td>
<td>(128, 64)</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>No. of taps, $P$</td>
<td>5</td>
</tr>
<tr>
<td>Delay profile ($\mu$s)</td>
<td>0.3, 1, 1.7, 2.4, 3.1</td>
</tr>
<tr>
<td>Doppler profile (Hz)</td>
<td>0, -400, 400, -1220, 1220</td>
</tr>
</tbody>
</table>
Figure: BER performance comparison between OTFS and OFDM systems with phase noise and Doppler shifts.

$f_c = 28$ GHz, $\Delta f = 78.125$ kHz

$M = 128$, $N = 64$, $P = 5$, BPSK
Conclusions

- **OTFS modulation**
  - an emerging and promising modulation scheme for high-Doppler fading channels
  - multiplexing in the delay-Doppler domain
  - impressive performance (superior performance compared to OFDM)
  - implementation using pre- and post-processing blocks to OFDM
  - simple channel estimation in the delay-Doppler domain
  - can serve in interesting use cases (high-speed trains, autonomous vehicles, drones, mmWave communications)

- Promising modulation scheme for 5G and beyond
Publications from our group


Thank you