Joint Channel Estimation and Soft-Symbol Detection in Massive MIMO Systems with Low Resolution ADCs

Chandra R. Murthy

February 22, 2020

Department of ECE, Indian Institute of Science, Bangalore, India
Joint work with **Sai Subramanyam Thoota**

Outline

Motivation & Approach

Motivation

- Massive MIMO: a key technology for 5G
- Large number of RF chains and ADCs
  - Power consumption of ADCs increases exponentially with bit-width

Motivation

- Massive MIMO: a key technology for 5G
- Large number of RF chains and ADCs
  - Power consumption of ADCs increases exponentially with bit-width
- Solution: use low resolution ADCs
  - Pro: energy efficient; lower silicon footprint
  - Cons: non-linearity due to quantization; large training overhead

Motivation

- Massive MIMO: a key technology for 5G
- Large number of RF chains and ADCs
  - Power consumption of ADCs increases exponentially with bit-width
- Solution: use low resolution ADCs
  - Pro: energy efficient; lower silicon footprint
  - Cons: non-linearity due to quantization; large training overhead
- Challenges:
  - Both pilots and data coarsely quantized
    - Perfect CSIR is an impractical assumption
    - Need joint channel estimation and symbol decoding
  - Soft-symbol decoding for coded communication

Approach

- We present a statistical interference approach to the channel estimation and soft symbol decoding problem

Approach

- We present a statistical interference approach to the channel estimation and soft symbol decoding problem
- Why statistical inference approach?

Approach

- We present a statistical interference approach to the channel estimation and soft symbol decoding problem
- Why statistical inference approach?
  - A great fit for coded communication systems
  - Computing marginal posterior distributions of the transmitted bits provide an elegant way to obtain the log likelihood ratios

Approach

- We present a statistical interference approach to the channel estimation and soft symbol decoding problem
- Why statistical inference approach?
  - A great fit for coded communication systems
  - Computing marginal posterior distributions of the transmitted bits provide an elegant way to obtain the log likelihood ratios
- Why not exact inference?

Approach

- We present a statistical interference approach to the channel estimation and soft symbol decoding problem
- Why statistical inference approach?
  - A great fit for coded communication systems
  - Computing marginal posterior distributions of the transmitted bits provide an elegant way to obtain the log likelihood ratios
- Why not exact inference?
  - Computational intractability
  - High dimensional integrals to compute the partition function

Approach

- We present a statistical interference approach to the channel estimation and soft symbol decoding problem
- Why statistical inference approach?
  - A great fit for coded communication systems
  - Computing marginal posterior distributions of the transmitted bits provide an elegant way to obtain the log likelihood ratios
- Why not exact inference?
  - Computational intractability
  - High dimensional integrals to compute the partition function
- Massive MIMO receiver: directed probabilistic graphical model
- Approximate inference of posterior distributions of data and channel
  - Tool: Variational Bayes inference

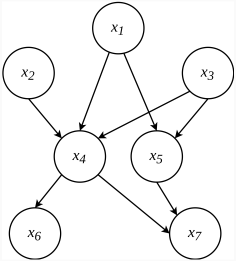
Variational Bayesian Inference

Overview of Probabilistic Graphical Models

- Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- $\mathcal{V}$ and $\mathcal{E}$ denote vertices (nodes) and edges, respectively
- Random variables or random vectors modeled as nodes in a graph
- Probabilistic dependence betn. random variables modeled as edges
- Examples: Bayesian networks, Markov random fields, factor graphs

Overview of Probabilistic Graphical Models

- Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- $\mathcal{V}$ and $\mathcal{E}$ denote vertices (nodes) and edges, respectively
- Random variables or random vectors modeled as nodes in a graph
- Probabilistic dependence betn. random variables modeled as edges
- Examples: Bayesian networks, Markov random fields, factor graphs
- Bayesian network:



- Joint distribution:

$$p(x_1)p(x_2)p(x_3)p(x_4|x_1, x_2, x_3)p(x_5|x_1, x_3)p(x_6|x_4)p(x_7|x_4, x_5)$$

Approximate Inference

- Main task in statistical inference: compute the posterior distribution $p(\mathbf{X}|\mathbf{Z})$ of the latent variables $\mathbf{X}$ given the observed data $\mathbf{Z}$

$$p(\mathbf{X}|\mathbf{Z}) = \frac{p(\mathbf{Z}|\mathbf{X})p(\mathbf{X})}{\int_{\mathbf{X}'} p(\mathbf{Z}|\mathbf{X}')p(\mathbf{X}')d\mathbf{X}'}$$

- Computing the partition function (denominator) is the bottleneck

- Main task in statistical inference: compute the posterior distribution $p(\mathbf{X}|\mathbf{Z})$ of the latent variables $\mathbf{X}$ given the observed data $\mathbf{Z}$

$$p(\mathbf{X}|\mathbf{Z}) = \frac{p(\mathbf{Z}|\mathbf{X})p(\mathbf{X})}{\int_{\mathbf{X}'} p(\mathbf{Z}|\mathbf{X}')p(\mathbf{X}')d\mathbf{X}'}$$

- Computing the partition function (denominator) is the bottleneck
- What is the way out?
    - Approximate the posterior
- Stochastic
    - Markov chain Monte Carlo sampling (e.g. Metropolis Hastings algorithm)
- Deterministic
    - Variational Bayes
    - Expectation maximization
    - Expectation propagation, etc.

Variational Bayesian (VB) Inference

- Iterative procedure to infer marginal distributions of latent variables
- Observations $\mathbf{Z} = \{\mathbf{z}_1, \ldots, \mathbf{z}_N\}$; latent variables $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$
- **Goal**: Find posterior distribution $p(\mathbf{X}|\mathbf{Z})$ and model evidence $p(\mathbf{Z})$
- Model Evidence:

$$\ln p(\mathbf{Z}) = \mathcal{L}(q) + \mathrm{KL}(q \,\|\, p)$$

where

$$\mathcal{L}(q) \triangleq \int q(\mathbf{X}) \ln \left\{ \frac{p(\mathbf{Z}, \mathbf{X})}{q(\mathbf{X})} \right\} d\mathbf{X}$$

$$\mathrm{KL}(q\|p) = -\int q(\mathbf{X}) \ln \left\{ \frac{p(\mathbf{X}|\mathbf{Z})}{q(\mathbf{X})} \right\} d\mathbf{X} \geq 0$$

- Here, $q(\mathbf{X})$ is arbitrary, and can be approximated and optimized

- Seek $q(\mathbf{X})$ to max. the **evidence lower bound (ELBO)** $\mathcal{L}(q)$
- **Factorized** posterior structure imposed on q

$$q(\mathbf{X}) = \prod_{i=1}^{N} q_i(\mathbf{X}_i)$$

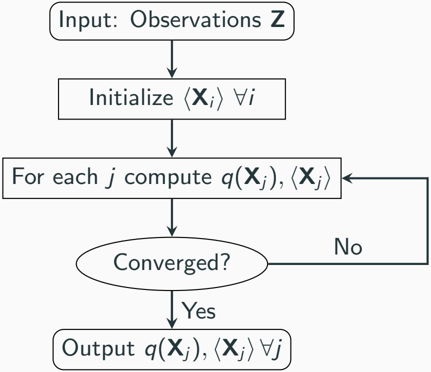
- After simplifying, the ELBO becomes

$$\mathcal{L}(q) = -\mathrm{KL}\left(q_j \| \tilde{p}(\mathbf{Z}, \mathbf{X}_j)\right) + \text{const.}$$

  where $\ln \tilde{p}(\mathbf{Z}, \mathbf{X}_j) \triangleq \mathbb{E}_{i \neq j}\left[\ln p(\mathbf{Z}, \mathbf{X})\right] + \text{const.}$ [1]
- To maximize $\mathcal{L}(q)$, minimize KL divergence
  - Minimum when $q_j(\mathbf{X}_j) = \tilde{p}(\mathbf{Z}, \mathbf{X}_j)$
- **Optimal** q_j is given by $\text{const} \times \exp\left(\mathbb{E}_{i \neq j}\left[\ln p(\mathbf{Z}, \mathbf{X})\right]\right)$
  - Fix $q_{i \neq j}$ and obtain the parameters of q_j and iterate for all j

[1] $\mathbb{E}_{i \neq j}\left[\ln p(\mathbf{Z}, \mathbf{X})\right] = \int \ln p(\mathbf{Z}, \mathbf{X}) \prod_{i \neq j} q_i(\mathbf{X}_i) d\mathbf{X}_i$

Flow Diagram



Joint Channel Estimation and Soft Symbol Decoding

Joint Channel Estimation and Soft Symbol Decoding

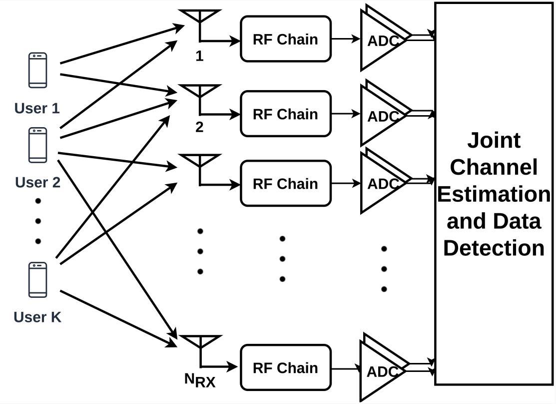
Goal

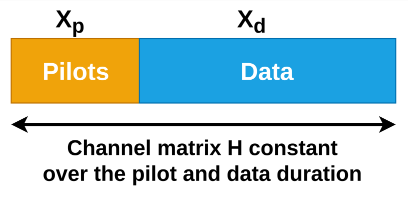
- Joint channel estimation and soft symbol decoding in the uplink of a massive MIMO system with low resolution ADCs

Contributions

- Posterior distributions of data symbols and channel inferred using a variational Bayesian inference framework
- Main algorithm developed: Quantized variational Bayes' joint channel estimator and soft symbol decoder
- Iterative algorithm which refines the channel and data estimates in each iteration

System Model



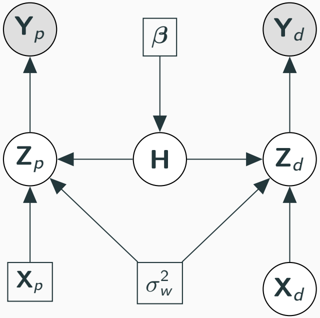


Received Signal

$$\mathbf{Y}_p = \mathcal{Q}(\mathbf{Z}_p) = \mathcal{Q}(\mathbf{H}\mathbf{X}_p + \mathbf{W}_p),$$
$$\mathbf{Y}_d = \mathcal{Q}(\mathbf{Z}_d) = \mathcal{Q}(\mathbf{H}\mathbf{X}_d + \mathbf{W}_d),$$

- $\mathcal{Q}(\cdot)$ denotes quantization operation

Bayesian Network Model



- Conditional probability distributions:

$$p\left(\mathbf{Z}_p|\mathbf{X}_p,\mathbf{H};\sigma_w^2\right)\propto\exp\left(-\frac{1}{\sigma_w^2}\sum_{t=1}^{\tau_p}\left\|\mathbf{z}_{p,t}-\mathbf{H}\mathbf{x}_{p,t}\right\|^2\right),$$

$$p\left(\mathbf{Z}_d|\mathbf{X}_d,\mathbf{H};\sigma_w^2\right)\propto\exp\left(-\frac{1}{\sigma_w^2}\sum_{t=1}^{\tau_d}\left\|\mathbf{z}_{d,t}-\mathbf{H}\mathbf{x}_{d,t}\right\|^2\right),$$

$$p\left(\mathbf{H}|\boldsymbol{\beta}\right)\propto\exp\left(-\sum_{k=1}^{K}\frac{1}{\beta_k}\left\|\mathbf{h}_k\right\|^2\right),$$

$$p\left(\mathbf{Y}_d|\mathbf{Z}_d\right)=\mathbb{1}\big(\mathbf{Z}_d\in[\mathbf{Z}_d^{(lo)},\mathbf{Z}_d^{(hi)}]\big),$$

$$p\left(\mathbf{Y}_p|\mathbf{Z}_p\right)=\mathbb{1}\big(\mathbf{Z}_p\in[\mathbf{Z}_p^{(lo)},\mathbf{Z}_p^{(hi)}]\big)$$

Quantized Variational Bayesian Joint Channel Estimation and Soft Symbol Decoding

Goal

- Infer the marginal posterior distribution of the channel and data symbols

Solution Approach

- Fully factorized approximation of the posterior distribution:

$$p\left(\mathbf{Z}_p, \mathbf{Z}_d, \mathbf{X}_d, \mathbf{H} | \mathbf{Y}_p, \mathbf{Y}_d, \mathbf{X}_p; \boldsymbol{\beta}, \sigma_w^2\right) \approx q\left(\mathbf{Z}_p\right) q\left(\mathbf{Z}_d\right) q\left(\mathbf{X}_d\right) q\left(\mathbf{H}\right),$$

where

$$q\left(\mathbf{H}\right) = \prod_{n=1}^{N_{RX}} \prod_{k=1}^{K} q\left(h_{nk}\right), q\left(\mathbf{X}_d\right) = \prod_{k=1}^{K} \prod_{t=1}^{\tau_d} q\left(x_{d,kt}\right),$$

$$q\left(\mathbf{Z}_d\right) = \prod_{n=1}^{N_{RX}} \prod_{t=1}^{\tau_d} q\left(z_{d,nt}\right), q\left(\mathbf{Z}_p\right) = \prod_{n=1}^{N_{RX}} \prod_{t=1}^{\tau_p} q\left(z_{p,nt}\right).$$

- **Computation of $q(h_{nk})$**:

$$\ln q\left(h_{nk}\right) \propto \left\langle \ln p\left(\mathbf{Z}_p | \mathbf{X}_p, \mathbf{H}; \sigma_w^2\right) + \ln p\left(\mathbf{Z}_d | \mathbf{X}_d, \mathbf{H}; \sigma_w^2\right) + \ln p\left(\mathbf{H} | \boldsymbol{\beta}\right) \right\rangle ,$$

$$\propto -\frac{1}{\sigma_w^2}\Bigg\{ \Bigg(\sum_{t=1}^{\tau_p} \left| x_{p,kt} \right|^2 + \sum_{t=1}^{\tau_d} \left\langle \left| x_{d,kt} \right|^2 \right\rangle + \frac{\sigma_w^2}{\beta_k} \Bigg) \left| h_{nk} \right|^2$$

$$- 2\Re\Bigg(\Bigg(\sum_{t=1}^{\tau_p} \Bigg[\left\langle z_{p,nt} \right\rangle^* x_{p,kt} - x_{p,kt} \sum_{\substack{k'=1 \\ k' \neq k}}^{K} x_{p,k't}^* \left\langle h_{nk'} \right\rangle^* \Bigg]$$

$$+ \sum_{t=1}^{\tau_d} \Bigg[\left\langle z_{d,nt} \right\rangle^* \left\langle x_{d,kt} \right\rangle - \left\langle x_{d,kt} \right\rangle \sum_{\substack{k'=1 \\ k' \neq k}}^{K} \left\langle x_{d,k't} \right\rangle^* \left\langle h_{nk'} \right\rangle^* \Bigg] \Bigg) h_{nk} \Bigg) \Bigg\}$$

- **Complex normal** distribution

- **Computation of $q(x_{d,kt})$:**

$$\ln q\left(x_{d,kt}\right) \propto \left\langle \ln p\left(\mathbf{Z}_d | \mathbf{H}, \mathbf{X}_d; \sigma_w^2\right) + \ln p\left(\mathbf{X}_d\right)\right\rangle,$$

$$\propto -\frac{1}{\sigma_w^2}\Bigg(\left\langle \|\mathbf{h}_k\|^2 \right\rangle |x_{d,kt}|^2 + 2\Re\Bigg[\Bigg(\sum_{\substack{k'=1 \\ k'\neq k}}^{K} \left\langle \mathbf{h}_{k'} \right\rangle^H \left\langle \mathbf{h}_k \right\rangle \left\langle x_{d,k't} \right\rangle^* - \left\langle \mathbf{z}_{d,t} \right\rangle^H \left\langle \mathbf{h}_k \right\rangle\Bigg) x_{d,kt}\Bigg]\Bigg).$$

- $x_{d,kt}$ takes values from a M-QAM constellation

$$q(x_{d,kt} = s) = \frac{\exp\left(-\frac{1}{\sigma_w^2} f(s)\right)}{\sum_{s'} \exp\left(-\frac{1}{\sigma_w^2} f(s')\right)},$$

where

$$f(s) = \left\langle \|\mathbf{h}_k\|^2 \right\rangle |s|^2 + 2\Re\Bigg[\Bigg(\sum_{\substack{k'=1 \\ k'\neq k}}^{K} \left\langle \mathbf{h}_{k'} \right\rangle^H \left\langle \mathbf{h}_k \right\rangle \left\langle x_{d,k't} \right\rangle^* - \left\langle \mathbf{z}_{d,t} \right\rangle^H \left\langle \mathbf{h}_k \right\rangle\Bigg) s\Bigg].$$

- **Boltzmann** distribution

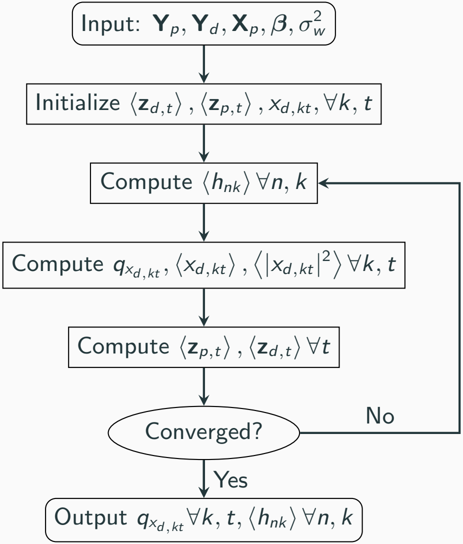
- **Computation of $q(\mathbf{z}_{d,t})$ and $q(\mathbf{z}_{p,t})$**:

$$\ln q\left(\mathbf{z}_{d,t}\right) \propto \left\langle \ln \mathbb{1}\left(\mathbf{z}_{d,t} \in [\mathbf{z}_{d,t}^{(lo)}, \mathbf{z}_{d,t}^{(hi)}]\right) - \frac{1}{\sigma_w^2} \left\|\mathbf{z}_{d,t} - \mathbf{H}\mathbf{x}_{d,t}\right\|^2 \right\rangle$$

- **Truncated complex normal distribution** with mean

$$\langle \mathbf{z}_{d,t} \rangle = \boldsymbol{\mu}_{\mathbf{z}_{d,t}} + \frac{\phi\left(\frac{\mathbf{z}_{d,t}^{(lo)} - \boldsymbol{\mu}_{\mathbf{z}_{d,t}}}{\sigma_w/\sqrt{2}}\right) - \phi\left(\frac{\mathbf{z}_{d,t}^{(hi)} - \boldsymbol{\mu}_{\mathbf{z}_{d,t}}}{\sigma_w/\sqrt{2}}\right)}{\Phi\left(\frac{\mathbf{z}_{d,t}^{(hi)} - \boldsymbol{\mu}_{\mathbf{z}_{d,t}}}{\sigma_w/\sqrt{2}}\right) - \Phi\left(\frac{\mathbf{z}_{d,t}^{(lo)} - \boldsymbol{\mu}_{\mathbf{z}_{d,t}}}{\sigma_w/\sqrt{2}}\right)} \frac{\sigma_w}{\sqrt{2}}$$

Flow Diagram of Joint Channel Estimator and Soft Symbol Decoder



Soft Symbol Decoding for MIMO-OFDM Systems

Soft Symbol Decoding for MIMO-OFDM Systems

Goal

- Soft symbol decoding in the uplink of a massive MIMO OFDM system with low resolution ADCs
- Assumptions:
  - Frequency selective fading
  - Perfect (or estimated) CSIR available

Contributions

- Posterior distributions of data symbols inferred using a variational Bayesian inference framework
- Main algorithm: Quantized variational Bayes' MIMO-OFDM soft symbol decoder

Received signal at the n_r^{th} antenna during the n^{th} symbol interval

$$z_{n_r}[n] = \sum_{k=1}^{K} \sum_{l=0}^{L-1} h_{n_r,k}[l]\overline{x}_k[n-l] + w_{n_r}[n], \qquad n = 0, \ldots, N_c - 1,$$

where

- $h_{n_r,k}[l]$ is the l^{th} tap of channel betn. the k^{th} UE and n_r^{th} RX antenna
- L is the length of the frequency selective channel
- $\overline{x}_k[m]$ is the data txed by the k^{th} UE in the m^{th} symbol duration
- N_c is the number of subcarriers
- w_{n_r} is the complex AWGN, distributed as $\mathcal{CN}(0, \sigma_w^2)$

- After CP removal and vectorization, the received signal is as follows:

$$\begin{aligned}\mathbf{Z}_{n_r} &= [z_{n_r}[0], z_{n_r}[1], \ldots, z_{n_r}[N_c - 1]]^T \\ &= \sum_{k=1}^{K} \mathbf{H}_{n_r,k}^{(t)} \bar{\mathbf{x}}_k + \mathbf{w}_{n_r} \\ &= \sum_{k=1}^{K} \mathbf{H}_{n_r,k}^{(t)} \mathbf{F}_{N_c}^H \mathbf{x}_k + \mathbf{w}_{n_r}\end{aligned}$$

  where $\mathbf{H}_{n_r,k}^{(t)} \in \mathbb{C}^{N_c \times N_c}$ is a circulant matrix with the first column as $[h_{n_r,k}[0], h_{n_r,k}[1], \ldots, h_{n_r,k}[L-1], \mathbf{0}_{N_c-L}^T]^T$

- Using the properties of circulant matrices, we get

$$\begin{aligned}\mathbf{Z}_{n_r} &= \sum_{k=1}^{K} \mathbf{F}_{N_c}^H \mathbf{H}_{n_r,k} \mathbf{x}_k + \mathbf{w}_{n_r} \\ &= (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H) \mathbf{H}_{n_r} \mathbf{x} + \mathbf{w}_{n_r},\end{aligned}$$

  where $\mathbf{H}_{n_r} = \mathrm{diag}(\mathbf{H}_{n_r,1}, \mathbf{H}_{n_r,2}, \ldots, \mathbf{H}_{n_r,K})$ is a diagonal matrix.

Quantized received signal

$$\mathbf{Y} = \mathcal{Q}\left(\left[\mathbf{Z}_1^T, \ldots, \mathbf{Z}_{N_r}^T\right]^T\right) \in \mathbb{C}^{N_r N_c \times 1}$$
$$= \mathcal{Q}\left(\begin{bmatrix} (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{H}_1 \\ (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{H}_2 \\ \vdots \quad \ddots \quad \vdots \\ (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{H}_{N_r} \end{bmatrix} \mathbf{x} + \mathbf{w}\right)$$
$$= \mathcal{Q}\left(\mathbf{H}\mathbf{x} + \mathbf{w}\right)$$

where

- $\mathbf{H} \in \mathbb{C}^{N_r N_c \times K N_c}$ is the composite channel
- $\mathbf{x} \in \mathbb{C}^{K N_c \times 1}$ is the data signal
- $\mathbf{w} \in \mathbb{C}^{N_r N_c \times 1}$ is the complex AWGN

Solution:

- Same as the flat fading case; can reuse previous solution
- For this part, assume $\mathbf{H}$ is known at the receiver

Iterative Channel Estimation and Soft Symbol Decoding for Massive MIMO OFDM Systems

Pilot and Data Transmission Model

Unquantized Rx signal at the n_r^{th} Rx antenna during the pilot and data phases

$$\mathbf{Z}_{n_r}^{(p)} = \sum_{k=1}^{K} \overline{\mathbf{X}}_k^{(p)} \overline{\mathbf{h}}_{n_r,k} + \mathbf{w}_{n_r}^{(p)} \in \mathbb{C}^{N_c \times 1}$$

$$\mathbf{Z}_{n_r}^{(d)}[t] = \sum_{k=1}^{K} \overline{\mathbf{X}}_k^{(d)}[t] \overline{\mathbf{h}}_{n_r,k} + \mathbf{w}_{n_r}^{(d)}[t] \in \mathbb{C}^{N_c \times 1},$$

where

- $\mathbf{h}_{n_r,k} = [h_{n_r,k}[0], \ldots, h_{n_r,k}[L-1]]^T \in \mathbb{C}^{L \times 1}$ is the k^{th} UE's channel
- $\overline{\mathbf{h}}_{n_r,k} = [\mathbf{h}_{n_r,k}^T, \mathbf{0}_{N_c-L}^T]^T \in \mathbb{C}^{N_c \times 1}$
- $\overline{\mathbf{X}}_k^{(p)} \in \mathbb{C}^{N_c \times N_c}$: circulant matrix with the first column as $\overline{\mathbf{x}}_k^{(p)}$
- $\overline{\mathbf{x}}_k^{(p)}$ and $\overline{\mathbf{x}}_k^{(d)}[t] \in \mathbb{C}^{N_c \times 1}$: IDFT of the k^{th} UE's pilot and t^{th} data symbol

Rewriting in matrix form and quantizing, we get the unquantized rx signal as

$$\mathbf{Z}^{(p)} = [\mathbf{Z}_1^{(p)}, \ldots, \mathbf{Z}_{N_r}^{(p)}] = (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{X}^{(p)}(\mathbf{I}_K \otimes \mathbf{F}_{N_c,L})\mathbf{H} + \mathbf{W}^{(p)}$$

$$\mathbf{Z}^{(d)}[t] = [\mathbf{Z}_1^{(d)}[t], \ldots, \mathbf{Z}_{N_r}^{(d)}[t]] = (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{X}^{(d)}[t](\mathbf{I}_K \otimes \mathbf{F}_{N_c,L})\mathbf{H} + \mathbf{W}^{(d)}[t], \quad \forall t$$

where

- $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{N_r}] \in \mathbb{C}^{KL \times N_r}$ is a row sparse matrix
- $\mathbf{X}^{(p)} = \text{diag}(\mathbf{X}_1^{(p)}, \ldots, \mathbf{X}_K^{(p)}) \in \mathbb{C}^{KN_c \times KN_c}$
- $\mathbf{X}^{(d)}[t] = \text{diag}(\mathbf{X}_1^{(d)}[t], \ldots, \mathbf{X}_K^{(d)}[t]) \in \mathbb{C}^{KN_c \times KN_c}$
- $\mathbf{F}_{N_c,L} \in \mathbb{C}^{N_c \times L}$ is the column truncated DFT matrix.

Quantized received signal in one coherence interval

$$\begin{aligned}
\mathbf{Y} &= [\mathbf{Y}^{(p)T}, \mathbf{Y}^{(d)T}[1], \ldots, \mathbf{Y}^{(d)T}[\tau_d]]^T \\
&= \mathcal{Q}([\mathbf{Z}^{(p)T}, \mathbf{Z}^{(d)T}[1], \ldots, \mathbf{Z}^{(d)T}[\tau_d]]^T) \\
&= \mathcal{Q}\left(\begin{bmatrix} (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{X}^{(p)}(\mathbf{I}_K \otimes \mathbf{F}_{N_c,L}) \\ (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{X}^{(d)}[1](\mathbf{I}_K \otimes \mathbf{F}_{N_c,L}) \\ \vdots \quad \ddots \quad \vdots \\ (\mathbf{1}_K^T \otimes \mathbf{F}_{N_c}^H)\mathbf{X}^{(d)}[\tau_d](\mathbf{I}_K \otimes \mathbf{F}_{N_c,L}) \end{bmatrix} \mathbf{H} + \mathbf{W}\right) \\
&= \mathcal{Q}(\mathbf{D}\mathbf{H} + \mathbf{W})
\end{aligned}$$

Iterative Channel Estimation and Soft Symbol Decoding

Goal

- To estimate the channel $\mathbf{H}$ and decode the data $\mathbf{X}^{(d)}[t]$, $t = 1, \ldots, \tau_d$ given the quantized received and pilot signals

Solution

- Iterative channel estimation and soft symbol decoding
- Initial quantized VB sparse channel estimation[2] using pilot observations
- Quantized VB soft symbol decoding (with estimated CSIR)
- Posterior mean of the decoded data symbols used to refine the channel estimates in subsequent iterations

---

[2] Y. Ding, S. E. Chiu, B. D. Rao, "Sparse recovery with quantized multiple measurement vectors", Asilomar Conference on Signals, Systems and Computers, 2017.

Variational Bayes Channel Estimation

System Model

$$\mathbf{Y} = \mathcal{Q}(\mathbf{Z}) = \mathcal{Q}(\mathbf{DH} + \mathbf{W})$$

- $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{N_r}]$
- Each entry of $\mathbf{W}$ distributed i.i.d. as $\mathcal{CN}(0, \sigma^2)$

VB Channel Estimation

- Fully factorized approximation of the posterior distribution

$$p(\mathbf{Z}, \mathbf{H}, \boldsymbol{\gamma} \,|\, \mathbf{Y}; \mathbf{D}, \sigma^2, a, b) \approx q_{\mathbf{Z}}(\mathbf{Z}) q_{\mathbf{H}}(\mathbf{H}) q_{\boldsymbol{\gamma}}(\boldsymbol{\gamma}),$$

  - Prior on $\mathbf{H}$

$$p(\mathbf{h}_l | \boldsymbol{\gamma}) = \mathcal{CN}(\mathbf{0}, \boldsymbol{\Gamma}^{-1}) \qquad \boldsymbol{\Gamma} = \mathrm{diag}(\boldsymbol{\gamma}),\ l = 1, \ldots N_r$$

    - Common $\boldsymbol{\gamma}$ across all l enforces row sparsity
  - Sparsity promoting hyperprior on $\boldsymbol{\gamma}$

$$p(\gamma_n) = \frac{b^a}{\Gamma(a)} \gamma_n^{a-1} \exp\{-b\gamma_n\}, \qquad \gamma_n > 0, \quad n = 1, \ldots, KL.$$

- Approximate posteriors obtained using VB procedure

Approximate posterior distributions

- $\mathbf{Z}$ is truncated complex Gaussian distributed with mean

$$\langle z_{ml} \rangle_{q(\mathbf{z}_l)} = [\mathbf{D} \langle \mathbf{h}_l \rangle_{q(\mathbf{h}_l)}]_m + \frac{\sigma}{\sqrt{2}} \cdot \frac{\phi(\alpha) - \phi(\delta)}{\Phi(\delta) - \Phi(\alpha)}, \quad m = 1, \ldots, N_c, l = 1, \ldots, N_r$$

where α and δ depend on the quantizer limits and $\langle \mathbf{h}_l \rangle_{q(\mathbf{h}_l)}$

- $\mathbf{h}_l$ is complex Gaussian distributed with mean $\langle \mathbf{h}_l \rangle_{q(\mathbf{h}_l)}$ and covariance matrix $\mathbf{\Sigma}$

$$\mathbf{\Sigma} = \left(\frac{1}{\sigma^2} \mathbf{D}^H \mathbf{D} + \langle \mathbf{\Gamma} \rangle_{q(\gamma)} \right)^{-1},$$

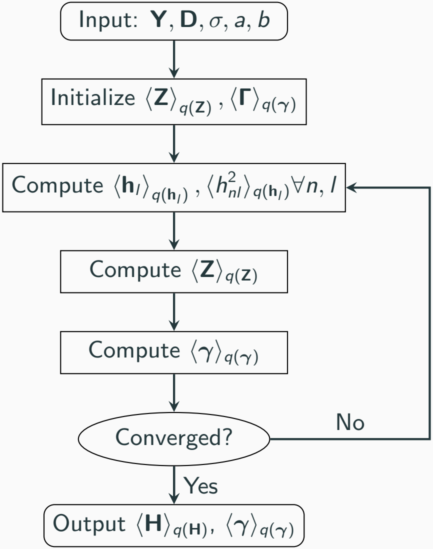
$$\langle \mathbf{h}_l \rangle_{q(\mathbf{h}_l)} = \frac{1}{\sigma^2} \mathbf{\Sigma} \mathbf{D}^H \langle \mathbf{z}_l \rangle_{q(\mathbf{z}_l)}, \qquad l = 1, \ldots, N_r.$$

- γ is Gamma distributed with mean

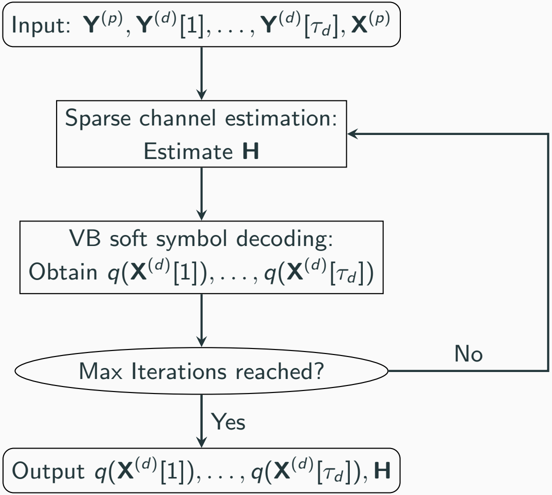
$$\langle \gamma_n \rangle_{q(\gamma_n)} = \frac{a + N_r}{b + \sum_{l=1}^{N_r} \langle |h_{nl}|^2 \rangle_{q(\mathbf{h}_l)}} \qquad n = 1, \ldots, KL.$$

  - $\langle |h_{nl}|^2 \rangle_{q(\mathbf{h}_l)}$ is the n^{th} element in $\text{diag}(\langle \mathbf{h}_l \rangle_{q(\mathbf{h}_l)} \langle \mathbf{h}_l \rangle_{q(\mathbf{h}_l)}^H + \mathbf{\Sigma})$

Flow Diagram of VB Channel Estimator



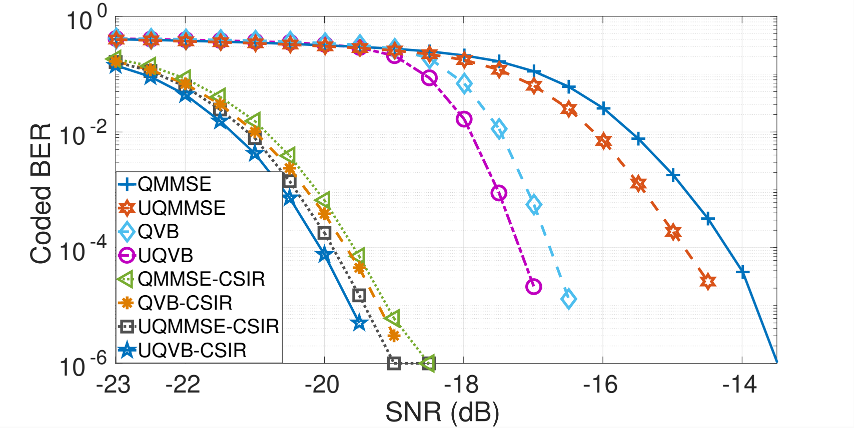
Flow Diagram of Iterative Channel Estimator and Soft Symbol Decoder



Simulation Results

Joint Channel Estimation and Soft Symbol Decoding[3]

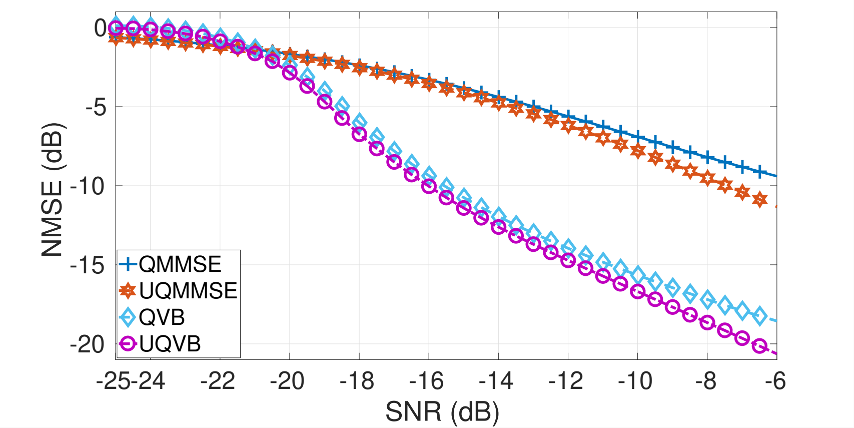
- $N_r = 200, K = 50, \tau_p = 50, \tau_d = 450$



- 3-bits quantization sufficient (≈ unquantized system)
- Perfect CSI assumption grossly overestimates the performance

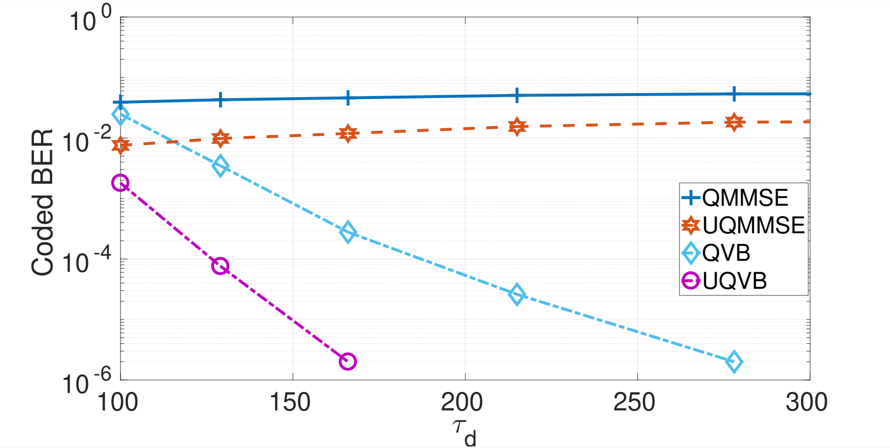
[3] 3GPP TS 38.212 v15.7.0 (2019-09), Tables 5.3.2-1, 5.3.2-2, 5.3.2-3, LDPC Code Rate = 1/3

NMSE Performance:



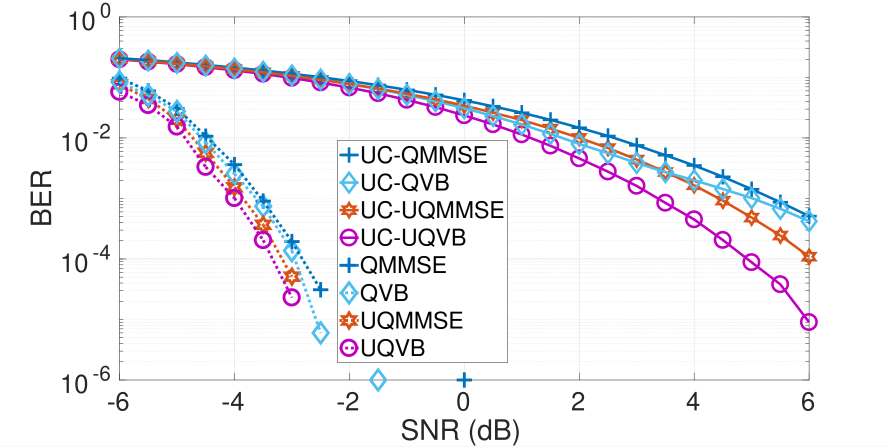
- Around 8 dB improvement at an NMSE of -10 dB compared to the MMSE estimation based on unquantized observations

- $N_r = 100$, $K = 25$, SNR=-13.5 dB, $\tau_p = 25$ and 3 bits quantization
- Effect of τ_d on the coded BER performance
  - As τ_d increases, performance improves



Soft Symbol decoding for MIMO-OFDM: Coded System Performance

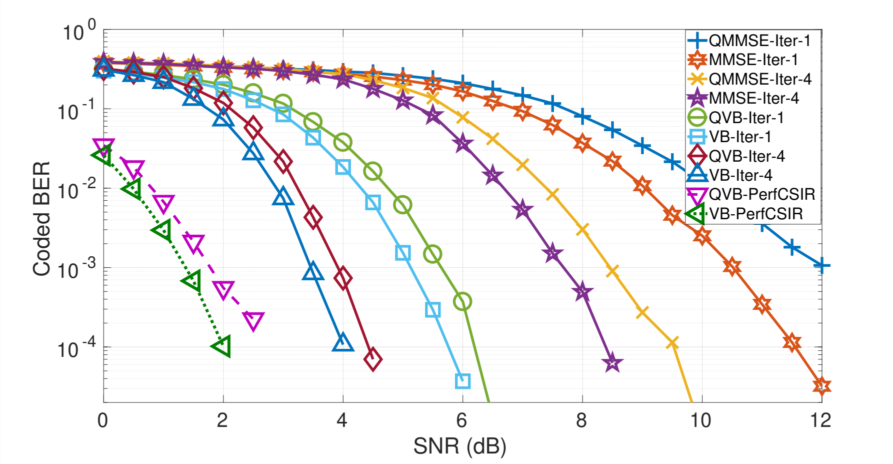
Simulation Setup: 16 users, 64 antennas, 128 subcarriers, 3−bits



- VB marginally outperforms conventional receiver under perfect CSIR

Simulation Setup

- Number of subcarriers $= 64$
- Number of receive antennas $= 32$
- Number of users $= 8$
- Channel length $= 8$
- Sparsity level $= 3$
- QPSK modulation
- Number of iterations of the receiver $= \{1, 4\}$
- Coherence interval $= \{5, 9\}$ OFDM symbols
- 3 bit quantization



- Performance improves with the number of iterations of the receiver
- Large improvement of QVB over QMMSE

Summary

Summary

- Proposed variational Bayesian inference algorithms for joint channel estimation and soft symbol decoding in uplink massive MIMO single carrier and OFDM systems with low resolution ADCs
- The algorithms output the posterior beliefs of the data symbols and channel estimates
- Iterative but computationally simple
- Guaranteed convergence to a local optimum

Publications

- S. S. Thoota, C. R. Murthy, "Variational Bayesian Inference based Soft-Symbol Decoding for Uplink Massive MIMO Systems with Low Resolution ADCs," Asilomar Conf. on Signals, Syst., and Comput., Pacific Grove, CA, USA, 2019.
- S. S. Thoota, C. R. Murthy, and R. Annavajjala, "Quantized Variational Bayesian Joint Channel Estimation and Data Detection for Uplink Massive MIMO Systems with Low Resolution ADCs," IEEE International Workshop on Machine Learning for Signal Processing (MLSP), Pittsburgh, PA, USA, 2019.
- S. S. Thoota and C. R. Murthy, "Variational Bayesian Channel Estimation and Soft Symbol Decoding for UL Massive MIMO OFDM Systems with Low Resolution ADCs," Submitted to IEEE JSAC Special Issue on Massive Access for 5G and beyond, Jan 2020.

Acknowledgments

- Young Faculty Research Fellowship, MeitY, Govt. of India
- Sponsored research project from Intel, Inc.
- Ramesh Annavajjala, Northeastern University
- Organizers of NCC 2020!

Thank You!

cmurthy@iisc.ac.in